Quasi-Nash Equilibria for Non-convex Distributed Power Allocation Games in Cognitive Radios

Xiaoge Huang, *Student Member, IEEE,* Baltasar Beferull-Lozano, *Senior Member, IEEE,*
Carmen Botella, *Member, IEEE*

***Abstract*—In this paper, we consider a sensing-based spectrum sharing scenario in cognitive radio networks where the overall objective is to maximize the sum-rate of each cognitive radio user by optimizing jointly both the detection operation based on sensing and the power allocation, taking into account the influence of the sensing accuracy and the interference limitation to the primary users. The resulting optimization problem for each cognitive user is non-convex, thus leading to a non-convex game, which presents a new challenge when analyzing the equilibria of this game where each cognitive user represents a player. In order to deal with the non-convexity of the game, we use a new relaxed equilibria concept, namely, quasi-Nash equilibrium (QNE). A QNE is a solution of a variational inequality obtained under the first-order optimality conditions of the player's problems, while retaining the convex constraints in the variational inequality problem. In this work, we state the sufficient conditions for the existence of the QNE for the proposed game. Specifically, under the so-called linear independent constraint qualification, we prove that the achieved QNE coincides with the NE. Moreover, a distributed primal-dual interior point optimization algorithm that converges to a QNE of the proposed game is provided in the paper, which is shown from the simulations to yield a considerable performance improvement with respect to an alternating direction optimization algorithm and a deterministic game.**

***Index Terms*—Cognitive radio, quasi-Nash equilibrium, non-convex game, variational inequality theory.**

I. Introduction

THe concept of cognitive radio (CR) has been proposed as a promising technology to improve spectrum utilization efficiency while limiting the performance degradation caused to primary users (PUs). The fundamental principle of a CR network (CRN) is to enhance the efficiency and flexibility in spectrum usage by allowing CR users to access the resources owned by PUs in an opportunistic manner [2]. There are currently three main approaches for cognitive communications regarding the way secondary users access the licensed spectrum: (i) opportunistic spectrum access (OSA) [3], where the CR user decides to access the channel only if the PU transmission is detected to be idle; (ii) spectrum sharing [4], where the CR user coexists with the PU and applies an interference constraint without sensing information to ensure the quality of service of the PU network; (iii) sensing-based spectrum sharing (SSS) [5], where the CR user senses the status of the channel and adapts its transmit power based on the decision made by spectrum sensing. In this paper, we consider the SSS scheme, where the CR transmitter deals with a performance tradeoff between maximizing its sum-rate and minimizing the performance degradation caused to the PU.

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A. Related Work

The problem of maximizing the rate of the CR user under perfect sensing information (e.g. the probability of miss detection and false alarm are zero) has been widely studied in the literature [6]–[8]. However, in practice, the reliability of the PU detection at the CR transmitter is limited by several factors, such as the attenuation due to path-loss, as well as shadowing and fading. As a consequence, a certain degree of performance degradation of the PU is usually unavoidable. In this case, the influence of the sensing accuracy on the rate of the CR user should be taken into account in order to perform an appropriate power allocation. Some previous works have focused on the combination of the sensing information with the rate of a simplified CRN with one CR user and one PU [9]–[12]. The authors of [9] consider the sensing-rate tradeoff for the OSA scheme assuming a single channel. The problem of designing the optimal sensing time and power allocation strategy that maximizes the average rate for the OSA and SSS schemes are studied in [10] and [11], respectively. The work in [11] is extended in [12], where the problem of finding the optimal sensing time and power allocation is studied based on the outage capacity constraint and the truncated channel inversion constraint, namely, a sensing-enhanced spectrum sharing CR system. All the aforementioned schemes are applicable only for a single CRN.

In a distributed multiuser scenario, CR users can self-enforce the negotiated agreements on the usage of the available spectrum. Every CR user aims at the transmission strategy that maximizes its own utility function, usually the average rate. This inherently competitive nature of the distributed multiuser scenario leads to a non-cooperative game (NCG) [13], where the solution of the game is the well-known concept of Nash equilibrium (NE). The NCG theoretical model for power allocation in the SISO and MIMO interference channels has been widely studied in [14]–[18], while the equilibrium model based on pricing has been discussed in [19] and [20]. However, the power allocation schemes proposed in the mentioned papers are not applicable to CR systems, since they do not provide any mechanism to limit the performance degradation caused to PUs. Recently, NCG theory has been successfully applied to the power allocation problem in CRNs [21]–[25]. The finite-dimensional variational inequality (VI) method [26] has been used in [21]–[24] to analyze the existence and uniqueness of the solution for the NCG in the CRN. Those works are extended in [25] for a more practical scenario with imperfect channel state information. However, in [21]–[25], no sensing is performed by CR users. The sensing information is considered in [27]–[29] for an OSA scenario, and the analysis of the equilibria of this game is based on a new concept called quasi-Nash equilibrium (QNE) [30]. QNE is a solution of a VI problem obtained under the first-order optimality conditions of each player's optimization problem, while retaining the convex constraints in the defining set of the VI problem. The prefix *quasi* is intended to signify that a NE must be a QNE under certain constraint qualifications (CQs) [30].

B. Contributions

In this paper, the resource allocation problem among CR users for the SSS scheme is analyzed as a strategic NCG, where each CR user is selfish and strives to use the available spectrum in order to maximize its own sum-rate by considering the effect of imperfect sensing information. The resulting game is non-convex due to non-convexity in both players' objective functions and constraints. Therefore, traditional mathematical tools are not applicable to show the existence of an equilibrium for this game. We analyze the non-convex non-cooperative power allocation game (NNPG) based on the new relaxed mathematical equilibria concept introduced in [30], the QNE. The main contributions of the paper are the following:

- We propose a NNPG, where each CR user aims at maximizing its own sum-rate by jointly optimizing the sensing information as well as the transmit power over all channels, which differs from the disjoint case, called deterministic game, as shown in [21]–[25].
- Deviating from the constraints considered in [9]–[12], [21]–[25], [27]–[29] (such as an interference temperature and outage probability constraints), a rate-loss constraint is introduced in order to effectively protect the PU from harmful interference caused due to the imperfect sensing information. The optimization problem is analyzed in two different limited regimes, namely, power budget limited regime and rate-loss limited regime. The performance of the CR users in these regimes are evaluated extensively through simulation.
- In addition, a distributed cooperative sensing scheme based on a consensus algorithm is considered in the proposed game for a SSS scenario. Compared with the OSA scenario discussed in [27]–[29], in the scenario, the CR users can coexist with PUs, and adjust the transmit power on each channel based on the sensing result (see Section II for details).
- The fourth major contribution of this paper is to prove that the proposed NNPG can achieve a QNE under certain conditions, by making use of the VI theory. Meanwhile, we show that, under the so-called linear independent constraint qualification, the achieved QNE coincides with the NE. Finally, an iterative primal-dual interior point (PDIP) algorithm that converges to a QNE of the proposed game is provided here. The PDIP algorithm can run at each node in parallel, since it requires only the local information of each CR user (e.g. its own transmit power and the channel gain), and hence, it can be regarded as a distributed solution. Simulation results show that the PDIP algorithm yields a considerable performance improvement, in terms of the sum-rate of each CR user, with respect to previous state-of-the-art methods, such as alternating direction optimization algorithm [1] and the deterministic game proposed in [25].

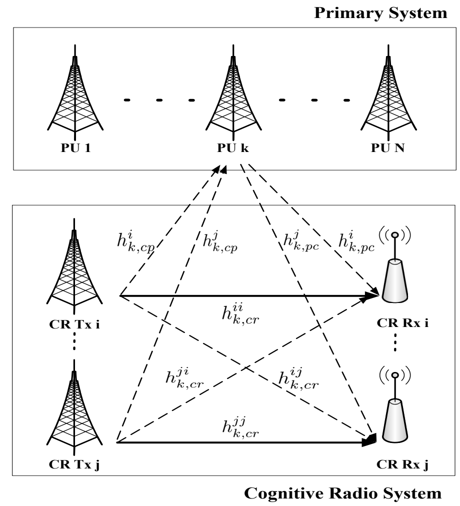

Fig. 1. System model: N PUs and M CR Tx-Rx pairs. PU k uses channel k, $k = 1, ..., N$.

The rest of the paper is organized as follows. Section II presents the system model. The analysis of the NNPG with imperfect sensing information is presented in Section III. The concept and the existence of a QNE is discussed in Section IV. Section V provides a detailed analysis of the primal-dual interior point optimization algorithm. Extensive performance evaluation results are presented in Section VI. Section VII states the conclusions.

Notation: Vectors and matrices are boldface, $[x_k]_{k=1}^N = [x_1, x_2, ..., x_N]$, $\nabla_{\mathbf{x}} f(\mathbf{x})$ denotes the gradient of function $f(\mathbf{x})$ at point $\mathbf{x}$, $\mathbf{J}_{f(\mathbf{x})}$ denotes the Jacobian matrix of the vector function $f(\mathbf{x})$, $Diag$ denotes the diagonal matrix, $\perp$ denotes "perpendicularity", $||\mathbf{x}||$ and $||\mathbf{x}||_\infty$ denote the Euclidean norm and the maximum norm of vector $\mathbf{x}$, respectively. $\mathbb{R}^n_+$ denotes the nonnegative n-dimensional space. P denotes power, $\mathcal{P}$ denotes probability. Tx and Rx denote transmitter and receiver, respectively. The variables $h^{ii}_{k,cr}$, $h^{ji}_{k,cr}$, $h^i_{k,cp}$ and $h^i_{k,pc}$ denote the instantaneous channel gains in channel k between CR-Tx i and CR-Rx i, CR-Tx j and CR-Rx i, CR-Tx i and PU, PU and CR-Rx i, respectively. We use CR i to indicate the ith CR pair.

II. System Model

Consider an OFDM-based communication system with N PUs, each one using a different channel (PU k uses channel k, $k=1,...,N$), and M CR Tx-Rx pairs which are close to each other. CR users are allowed to access the N channels simultaneously, thus the interference in a given channel is due to the interaction of the CR users (see Fig. 1). Before accessing the channel, each CR-Tx must first perform spectrum sensing to determine the status of each channel. In this paper, we assume that simultaneous spectrum sensing of all the N channels is performed by each CR-Tx using an energy detection scheme. The detection problem on each channel is modeled as a hypothesis test, where hypothesis $H_{0,k}$ represents the absence of a PU in channel k, and the alternative hypothesis $H_{1,k}$ represents the presence of a PU in channel k. Specifically, for channel k, at the discrete sample l, the received signal y_k^i at the CR-Rx i, $i=1,2,...,M$, is given by [31]:

$$H_{0,k}: \quad y_k^i(l) = n_k(l) \tag{1}$$

$$H_{1,k}: \quad y_k^i(l) = S_k^i(l) + n_k(l) \tag{2}$$

where $n_k(l)$ denotes additive background noise on the k-th channel, which is assumed to be independent and identically distributed additive complex Gaussian with zero mean and variance $(\sigma_{k,n}^i)^2$, and $S_k^i(l)$ stands for the PU transmit signal in channel k. Let $P_{k,pc}^i = |S_k|^2|h_{k,pc}^i|^2$ denote the received power by CR-Rx i from the PU in channel k, and $L_s = tf_s$ denote the number of samples, where t is the sensing time and f_s represents the sampling frequency. Under an energy detection scheme, for each channel k, the decision is based on the sum of the received energy over an interval of its samples, that is, $Y_k = \sum_{l=1}^{L_s} |y_k^i(l)|^2$. Note that the longer the sensing time t, the better the energy estimation accuracy. However, for a fixed frame length, with a longer sensing time t, the transmission time has to be reduced. In order to improve the sensing accuracy without increasing sensing time t, a distributed cooperative scheme is adopted here. We assume that the nearby CR-Rxs have the possibility to exchange their local measurements, thus the cooperative sensing can be implemented by the distributed consensus algorithm from [32], which requires only the interaction among nearby CR-Rxs. Let us denote by M the number of cooperative CR-Rxs. State update occurs at discrete time for each CR-Rx locally, and the final average consensus result $y_{k,c}^i(l) = \frac{1}{M}\sum_{i=1}^{M} y_k^i(l)$ is asymptotically reached for all nodes [32]. The final sensing decision at each CR-Tx is made by comparing the consensus result with a primary detection threshold τ_k^i as follows [33]:

$$Y_{k,c} = \sum_{l=1}^{L_s} |y_{k,c}^i(l)|^2 \gtrless_{H_{0,k}}^{H_{1,k}} \tau_k^i, \quad k=1,2,\ldots,N. \tag{3}$$

According to the Central Limit Theorem, for large L_s, $y_{k,c}^i(l)$ are approximately normally distributed: $Y_{k,c} \sim \mathcal{N}(\mu_{k,0}^i, (\sigma_{k,0}^i)^2)$ for $H_{0,k}$, and $Y_{k,c} \sim \mathcal{N}(\mu_{k,1}^i, (\sigma_{k,1}^i)^2)$ for $H_{1,k}$, where:

$$\mathcal{N}(\mu_{k,0}^i, (\sigma_{k,0}^i)^2) \begin{cases} \mu_{k,0}^i = \frac{L_s}{M}\sum_{i=1}^{M}(\sigma_{k,n}^i)^2 \\ (\sigma_{k,0}^i)^2 = \frac{L_s}{M^2}\sum_{i=1}^{M}(\sigma_{k,n}^i)^4 \end{cases} \tag{4}$$

TABLE I

Four instantaneous rates at CR-Rx i

Actual status	Detection result	Actual rate at CR-Rx i
$H_{0,k}$	$H_{0,k}$	$r_{k,00}^i = \log_2\left(1+\frac{P_{k,0}^i\lvert h_{k,cr}^{ii}\rvert^2}{I_{k,0}^i}\right)$
$H_{0,k}$	$H_{1,k}$	$r_{k,01}^i = \log_2\left(1+\frac{P_{k,1}^i\lvert h_{k,cr}^{ii}\rvert^2}{I_{k,1}^i}\right)$
$H_{1,k}$	$H_{0,k}$	$r_{k,10}^i = \log_2\left(1+\frac{P_{k,0}^i\lvert h_{k,cr}^{ii}\rvert^2}{I_{k,0}^i+P_{k,pc}^i}\right)$
$H_{1,k}$	$H_{1,k}$	$r_{k,11}^i = \log_2\left(1+\frac{P_{k,1}^i\lvert h_{k,cr}^{ii}\rvert^2}{I_{k,1}^i+P_{k,pc}^i}\right)$

$$\mathcal{N}(\mu_{k,1}^i, (\sigma_{k,1}^i)^2) \begin{cases} \mu_{k,1}^i = \frac{L_s}{M}\sum_{i=1}^{M}((\sigma_{k,n}^i)^2 + P_{k,pc}^i) \\ (\sigma_{k,1}^i)^2 = \frac{L_s}{M^2}\sum_{i=1}^{M}((\sigma_{k,n}^i)^2 + P_{k,pc}^i)^2 \end{cases} \tag{5}$$

The probabilities of detection $\mathcal{P}_{k,d}^i$ and false alarm $\mathcal{P}_{k,fa}^i$ for the k-th channel for CR-Tx i, $i=1,2,...,M$, are given by:

$$\mathcal{P}_{k,fa}^i(\tau_k^i, t) = \mathcal{Q}\left(\frac{\tau_k^i - \mu_{k,0}^i}{\sigma_{k,0}^i}\right) \tag{6}$$

$$\mathcal{P}_{k,d}^i(\tau_k^i, t) = \mathcal{Q}\left(\frac{\tau_k^i - \mu_{k,1}^i}{\sigma_{k,1}^i}\right) \tag{7}$$

In this paper, we consider a SSS scheme, where CR-Txs transmit simultaneously on the N channels and adapt their transmit power on each channel based on the sensing information. If channel k is detected to be idle $(H_{0,k})$, CR-Tx i transmits using power $P_{k,0}^i$, whereas if channel k is sensed to be active $(H_{1,k})$, then each CR-Tx i transmits using a relatively lower power $P_{k,1}^i$, in order to reduce the interference caused to the PU. This scheme can be seen as a hybrid approach between protecting the PU and improving the spectrum utilization.

III. Joint Optimization of Detection and Power Allocation

Spectral efficiency is the main overall goal of the CR users, thus the objective function chosen by each user to be maximized is the sum-rate over all the channels. In this section, we analyze the problem of optimizing the power allocation for the CR users in order to maximize the sum-rate, taking into account the detection result. Considering the fact that the spectrum sensing information is not always reliable, which implies having probabilities of detection $\mathcal{P}_{k,d}^i < 1$ and probabilities of false alarm $\mathcal{P}_{k,fa}^i > 0$, we have four different instantaneous rates at CR-Rx i in channel k, as shown in Table I. In this table, the first subindex number of r_k^i (the third column of Table I) describes the actual status of the PU ("0" for idle and "1" for active), and the second subindex number indicates the sensing result obtained by energy detection. $I_{k,0}^i$ and $I_{k,1}^i$, presenting the noise and the interference observed by CR-Rx i from other CR-Txs in channel k, under sensing results $H_{0,k}$ and $H_{1,k}$ are given by:

$$I_{k,0}^i = (\sigma_{k,n}^i)^2 + \sum_{j=1, j\neq i}^{M} P_{k,0}^i|h_{k,cr}^{ji}|^2 \tag{8}$$

$$I_{k,1}^i = (\sigma_{k,n}^i)^2 + \sum_{j=1, j\neq i}^{M} P_{k,1}^i|h_{k,cr}^{ji}|^2 \tag{9}$$

Let $\mathcal{P}(H_{0,k})$ denote the prior probability that the k-th channel is idle, and $\mathcal{P}(H_{1,k})$ denote the prior probability that the k-th channel is active. The total achievable average rate at CR-Rx i based on a given sensing time t, denoted as $f^i(\mathbf{P}_1^i, \mathbf{P}_0^i, \boldsymbol{\tau}^i)$, $\mathbf{P}_1^i = [P_{k,1}^i]_{k=1}^N$, $\mathbf{P}_0^i = [P_{k,0}^i]_{k=1}^N$, $\boldsymbol{\tau}^i = [\tau_k^i]_{k=1}^N$, can be formulated as follows:

$$\sum_{k=1}^{N}\Bigg(\mathcal{P}(H_{0,k})\Bigg((1-\mathcal{P}_{k,fa}^i(\tau_k^i))r_{k,00}^i + \mathcal{P}_{k,fa}^i(\tau_k^i)r_{k,01}^i\Bigg) + \mathcal{P}(H_{1,k})\Bigg((1-\mathcal{P}_{k,d}^i(\tau_k^i))r_{k,10}^i + \mathcal{P}_{k,d}^i(\tau_k^i)r_{k,11}^i\Bigg)\Bigg) \quad (10)$$

The most important constraint of a CRN involves protecting the PU from harmful performance degradation. This constraint can be imposed on an individual or global level. The individual constraint requires the transmit power of each CR user in channel k to be always less than a given threshold. Instead of specifying individual constraints on the transmit power of each CR user per channel, the global constraint adapts the transmit power of each CR-Tx depending on the actions from other CR users that share the same channel, so that the accumulated interference from all the CR users at a PU does not exceed a given threshold. Though the global constraint may result in higher network rate (with price mechanism), it requires a large information exchange and coordination among CR users, as shown in [28], [29]. In this paper, we assume that the CR users are not willing to exchange information. Therefore, we use an individual constraint, namely, rate-loss constraint, to design the power allocation, ensuring that the performance degradation experienced by each PU is bounded. This individual constraint leads to a distributed scenario (see Sec.V). Note that only local information exchange among nearby CR-Rxs is needed in the cooperative sensing stage. On the one hand, the maximum achievable average rate of the PU in channel k without the interference from CR-Tx i is denoted as:

$$R_{k,\max}^i = \mathcal{P}(H_{1,k})\log_2\left(1+\frac{|S_k|^2}{(\sigma_{k,n}^i)^2}\right) \quad (11)$$

On the other hand, the maximum achievable average rate of the PU in channel k with the interference from CR-Tx i is denoted as:

$$R_k^i = \mathcal{P}(H_{1,k})\mathcal{P}_{k,d}^i\log_2\left(1+\frac{|S_k|^2}{I_{k,1}^{i,p}}\right) + \mathcal{P}(H_{1,k})(1-\mathcal{P}_{k,d}^i)\log_2\left(1+\frac{|S_k|^2}{I_{k,0}^{i,p}}\right) \quad (12)$$

where $I_{k,0}^{i,p}$ and $I_{k,0}^{i,p}$ are the noise and the interference from CR-Tx i to the PU in channel k under sensing results $H_{0,k}$ and $H_{1,k}$, respectively:

$$I_{k,0}^{i,p} = (\sigma_{k,n}^i)^2 + P_{k,0}^i|h_{k,cp}^i|^2 \quad (13)$$

$$I_{k,1}^{i,p} = (\sigma_{k,n}^i)^2 + P_{k,1}^i|h_{k,cp}^i|^2 \quad (14)$$

Let Γ_k denote the maximum acceptable rate-loss gap of the PU in channel k, $k=1,...,N$. Then, the rate-loss constraint for CR-Tx i can be written as follows:

$$R_{k,\max}^i - R_k^i \le \Gamma_k R_{k,\max}^i \quad (15)$$

In order to simplify the development of (15), we use $x\log_2(e)$ instead of $\log_2(1+x)$, which amounts to rewrite this constraint as:

$$(1-\Gamma_k)\mathcal{P}(H_{1,k})\frac{|S_k|^2\log_2 e}{(\sigma_{k,n}^i)^2} - \mathcal{P}(H_{1,k})\mathcal{P}_{k,d}^i\frac{|S_k|^2\log_2 e}{I_{k,1}^{i,p}} - \mathcal{P}(H_{1,k})(1-\mathcal{P}_{k,d}^i)\frac{|S_k|^2\log_2 e}{I_{k,0}^{i,p}} \le 0 \quad (16)$$

Given this, the new rate-loss constraint results in:

$$\Gamma_{k,c}^i I_{k,1}^{i,p} I_{k,0}^{i,p} - \mathcal{P}_{k,d}^i I_{k,0}^{i,p} - (1-\mathcal{P}_{k,d}^i)I_{k,1}^{i,p} \le 0 \quad (17)$$

where $\Gamma_{k,c}^i = (1-\Gamma_k)/(\sigma_{k,n}^i)^2$. In fact, since $x\log_2 e \ge \log_2(1+x)$, the actual rate-loss gap resulting from the constraint (17) is not the same as in the original constraint (15). The modified constraint (17) is more restrictive than (15), as shown in Sec. VI. The resulting solutions are valid and satisfactory, providing the sum-rate as the original constraint (15) obtained with a smaller rate-loss gap. Further details are given in Appendix A. Furthermore, the total transmit power of each CR-Tx i over all channels should not exceed its maximum allowed power. The power budget constraint for each CR-Tx i can be formulated as:

$$\sum_{k=1}^{N}\Bigg(\mathcal{P}(H_{0,k})((1-\mathcal{P}_{k,fa}^i(\tau_k^i))P_{k,0}^i + \mathcal{P}_{k,fa}^i(\tau_k^i)P_{k,1}^i) + \mathcal{P}(H_{1,k})(\mathcal{P}_{k,d}^i(\tau_k^i)P_{k,1}^i + (1-\mathcal{P}_{k,d}^i(\tau_k^i))P_{k,0}^i)\Bigg) \le P_{\max}^i \quad (18)$$

where $P_{\max}^i$ denotes the maximum total transmit power of the CR-Tx i over all the N channels. In a real system, a high $\mathcal{P}_{k,d}^i$ and a low $\mathcal{P}_{k,fa}^i$ are typically required. In this work, without loss of generality, we restrict the target detection probability and false alarm to the ranges $\mathcal{P}_{k,d}^i \ge \frac{1}{2}$ and $\mathcal{P}_{k,fa}^i \le \frac{1}{2}$, respectively. According to the monotonicity of the $\mathcal{Q}$-function, and taking into account (6) and (7), constraints in $\mathcal{P}_{k,d}^i$ and $\mathcal{P}_{k,fa}^i$ are equivalent to the inequalities:

$$\tau_{k,min}^i \le \tau_k^i \le \tau_{k,max}^i \quad (19)$$

where $\tau_{k,min}^i = \mu_{k,0}^i$, $\tau_{k,max}^i = \mu_{k,1}^i$. Finally, the optimization problem for maximizing the sum-rate of CR i can be formulated as the following problem $P1$:

$$\begin{aligned}
&\max_{\mathbf{P}_1^i, \mathbf{P}_0^i, \boldsymbol{\tau}^i} f^i(\mathbf{P}_1^i, \mathbf{P}_0^i, \boldsymbol{\tau}^i)\\
&\text{s. t. } (a1)\, \Gamma_{k,c}^i I_{k,1}^{i,p} I_{k,0}^{i,p} - \mathcal{P}_{k,d}^i I_{k,0}^{i,p} - (1-\mathcal{P}_{k,d}^i)I_{k,1}^{i,p} \le 0,\\
&\quad (a2) \sum_{k=1}^{N}(\mathcal{P}(H_{0,k})((1-\mathcal{P}_{k,fa}^i(\tau_k^i))P_{k,0}^i + \mathcal{P}_{k,fa}^i(\tau_k^i)P_{k,1}^i)\\
&\quad + \mathcal{P}(H_{1,k})(\mathcal{P}_{k,d}^i(\tau_k^i)P_{k,1}^i + (1-\mathcal{P}_{k,d}^i(\tau_k^i))P_{k,0}^i)) \le P_{\max}^i,\\
&\quad (b1)\, \tau_{k,min}^i \le \tau_k^i \le \tau_{k,max}^i,\\
&\quad (b2)\, 0 \le P_{k,1}^i, 0 \le P_{k,0}^i, 1 \le i \le M, 1 \le k \le N. \quad (20)
\end{aligned}$$

IV. QNE of the Non-convex Non-cooperative Power Allocation Game

In the scenario, we consider that CR users are selfish and strive to maximize their own sum-rate under several constraints, leading to a non-cooperative power allocation game.

TABLE II
NOTATION OF THE NON-CONVEX NON-COOPERATIVE POWER ALLOCATION GAME

Symbol	Meaning
$\boldsymbol{\tau}^i$	Detection threshold of CR i
$\mathbf{P}_0^i$	Transmit power of CR i for detection result H_0
$\mathbf{P}_1^i$	Transmit power of CR i for detection result H_1
$\mathbf{x}^i = (\mathbf{P}_0^i, \mathbf{P}_1^i, \boldsymbol{\tau}^i)$	Strategy set of CR i
$f^i(\mathbf{x}^i)$	Sum-rate of CR i
$g_k^i(\mathbf{x}^i)$	Non-convex individual constraint $(a1)$ of $P1$
$h^i(\mathbf{x}^i)$	Non-convex individual constraint $(a2)$ of $P1$
$\mathbf{J}_{g_k^i(\mathbf{x}^i)}$	Jacobian matrix of the vector function $g_k^i(\mathbf{x}^i)$
$\mathbf{J}_{h^i(\mathbf{x}^i)}$	Jacobian matrix of the vector function $h^i(\mathbf{x}^i)$
$\nabla^2_{\mathbf{x}^i} g_k^i(\mathbf{x}^i)$	Hessian matrix of the vector function $g_k^i(\mathbf{x}^i)$
$\nabla^2_{\mathbf{x}^i} h^i(\mathbf{x}^i)$	Hessian matrix of the vector function $h^i(\mathbf{x}^i)$
$T(\mathcal{X}^i; \mathbf{x}^i)$	Tangent cone of the set $\mathcal{X}^i$ at $\mathbf{x}^i \in \mathcal{X}^i$
$\mathcal{X}^i$	Convex individual constraints $(b1), (b2)$ of $P1$
$\mathcal{Y}^i$	Feasible set of CR i

Consider that there are M players, corresponding to the M CR-Txs, each one controlling the variables $\mathbf{x}^i = (\mathbf{P}_1^i, \mathbf{P}_0^i, \boldsymbol{\tau}^i)$. We denote by $\mathbf{x}$ the overall vector of all variables: $\mathbf{x} = [\mathbf{x}^i]_{i=1}^M$, while $\mathbf{x}^{-i} = (\mathbf{x}^1, ..., \mathbf{x}^{i-1}, \mathbf{x}^{i+1}, ..., \mathbf{x}^M)$ stands for the vector of the variables associated to all CR users except CR i. The main symbols used in this section are given in Table II.

The non-convex individual constraints $(a1)$ and $(a2)$ are denoted as $g_k^i(\mathbf{x}^i)$. We define the function vectors $\mathbf{G}(\mathbf{x}) = [(g_k^i(\mathbf{x}^i))_{k=1}^N]_{i=1}^M$, and $h^i(\mathbf{x}^i)$, $\mathbf{H}(\mathbf{x}) = [h^i(\mathbf{x}^i)]_{i=1}^M$, respectively, whereas the convex individual constraints $(b1), (b2)$ are embedded in the defining set of $\mathbf{x}^i$, denoted as $\mathcal{X}^i$. We denote the non-cooperative power allocation game $\mathcal{G}(\mathbf{H}, \mathbf{G})$, given as problem $P2$:

$$\begin{aligned} \max_{\mathbf{x}^i} \quad & f^i(\mathbf{x}^i) \\ \text{s. t.} \quad & g_k^i(\mathbf{x}^i) \leq 0, \;\; h^i(\mathbf{x}^i) \leq 0, \;\; \mathbf{x}^i \in \mathcal{X}^i. \end{aligned} \tag{21}$$

The resulting game $P2$ is non-convex; the objective function and the constraints are non-convex due to the presence of the false alarm and detection probabilities. As a consequence, traditional mathematical tools are not applicable to prove the existence of a NE for the game. In this section, we analyze the proposed non-convex game based on a relaxed equilibrium concept that has been recently introduced by Pang and Scutari [27], [28], namely, the quasi-Nash equilibrium (QNE).

A. Quasi-Nash equilibrium

We use the concept of QNE for the non-convex game $P2$, where the QNE is by definition a tuple that satisfies the Karush-Kuhn-Tucker (KKT) conditions of all the players' optimization problems; the prefix *quasi* is intended to signify that a NE (if it exists) must be a QNE under a certain constraint qualification (CQ), as explained in [27], [28]. Notice that for a nonlinear program constrained by finite equations and inequalities and a differentiable objective function, KKT conditions are not always necessary conditions for a given point to be a solution to the problem. When an appropriate CQ holds, the solutions of the KKT conditions are equal to stationary solutions of the associated problem [29]. In the following, the KKT conditions of the problem $P2$ are rewritten to a proper variational inequality (VI) problem [26]. Let $\mathcal{Y}^i$ denote the feasible strategy set of each CR i, which can be written as:

$$\mathcal{Y}^i = \{\mathbf{x}^i \in \mathcal{X}^i \mid g_k^i(\mathbf{x}^i) \leq 0, h^i(\mathbf{x}^i) \leq 0\}, \; 1 \leq k \leq N. \tag{22}$$

Instead of explicitly accounting all the multipliers as variables of the KKT conditions for each player's optimization problem, we introduce multipliers only for the non-convex constraints $h^i(\mathbf{x}^i) \leq 0$ and $g_k^i(\mathbf{x}^i) \leq 0$, while the convex constraints are embedded in the defining set $\mathcal{X}^i$. Denoting by α_k^i and β^i the multipliers associated with the non-convex constraints $g_k^i(\mathbf{x}^i) \leq 0$ and $h^i(\mathbf{x}^i) \leq 0$ of player CR-Tx i, respectively, the Lagrangian function of player CR-Tx i is given by:

$$L^i(\mathbf{x}^i, \boldsymbol{\alpha}^i, \boldsymbol{\beta}^i) = -f^i(\mathbf{x}^i) + \sum_{k=1}^N \alpha_k^i g_k^i(\mathbf{x}^i) + \beta^i h^i(\mathbf{x}^i) \tag{23}$$

The KKT conditions based on the Lagrangian function (23) are given by:

$$\begin{aligned} & 0 \leq \mathbf{x}^i \perp \nabla_{\mathbf{x}^i} L(\mathbf{x}^i, \boldsymbol{\alpha}^i, \boldsymbol{\beta}^i) \geq 0 \\ & 0 \leq \alpha_k^i \perp -g_k^i(\mathbf{x}^i) \geq 0 \\ & 0 \leq \beta^i \perp -h^i(\mathbf{x}^i) \geq 0 \end{aligned} \tag{24}$$

where $0 \leq a \perp b \geq 0$ implies $a \geq 0$, $b \geq 0$, $a \cdot b = 0$, and $\nabla_{\mathbf{x}^i} L(\mathbf{x}^i, \boldsymbol{\alpha}^i, \boldsymbol{\beta}^i)$ is defined as:

$$\nabla_{\mathbf{x}^i} L(\mathbf{x}^i, \boldsymbol{\alpha}^i, \boldsymbol{\beta}^i) \;\; = -\nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i) + \sum_{k=1}^N \alpha_k^i \mathbf{J}_{g_k^i(\mathbf{x}^i)} + \beta^i \mathbf{J}_{h^i(\mathbf{x}^i)}$$

The components of the gradient $\nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i) = (\nabla_{P_{k,0}^i} f^i(\mathbf{x}^i), \nabla_{P_{k,1}^i} f^i(\mathbf{x}^i), \;\; \nabla_{\tau_k^i} f^i(\mathbf{x}^i))$ are given, respectively, by:

$$\nabla_{P_{k,0}^i} f^i(\mathbf{x}^i) = \frac{a_{k,0}^i |h_{k,cr}^{ii}|^2}{I_{k,0}^i + P_{k,0}^i |h_{k,cr}^{ii}|^2} + \frac{b_{k,0}^i |h_{k,cr}^{ii}|^2}{P_{k,pc}^i + I_{k,0}^i + P_{k,0}^i |h_{k,cr}^{ii}|^2} \tag{25}$$

$$\nabla_{P_{k,1}^i} f^i(\mathbf{x}^i) = \frac{a_{k,1}^i |h_{k,cr}^{ii}|^2}{I_{k,1}^i + P_{k,1}^i |h_{k,cr}^{ii}|^2} + \frac{b_{k,1}^i |h_{k,cr}^{ii}|^2}{P_{k,pc}^i + I_{k,1}^i + P_{k,1}^i |h_{k,cr}^{ii}|^2} \tag{26}$$

$$\begin{aligned} \nabla_{\tau_k^i} f^i(\mathbf{x}^i) = & \; \mathcal{P}(H_{0,k}) \mathcal{P}_{k,fa}^i(\tau_k^i)'(r_{k,01}^i - r_{k,00}^i) \\ & + \mathcal{P}(H_{1,k}) \mathcal{P}_{k,d}^i(\tau_k^i)'(r_{k,11}^i - r_{k,10}^i) \end{aligned} \tag{27}$$

where:

$$\begin{aligned} & a_{k,0}^i = \mathcal{P}(H_{0,k})(1 - \mathcal{P}_{k,fa}^i(\tau_k^i)), \; a_{k,1}^i = \mathcal{P}(H_{0,k}) \mathcal{P}_{k,fa}^i(\tau_k^i) \\ & b_{k,0}^i = \mathcal{P}(H_{1,k})(1 - \mathcal{P}_{k,d}^i(\tau_k^i)), \;\; b_{k,1}^i = \mathcal{P}(H_{1,k}) \mathcal{P}_{k,d}^i(\tau_k^i) \end{aligned} \tag{28}$$

The components $\mathbf{J}_{g_k^i(\mathbf{x}^i)}$ and $\mathbf{J}_{h^i(\mathbf{x}^i)}$ denote the Jacobian matrix of the vector function $g_k^i(\mathbf{x}^i)$ and $h^i(\mathbf{x}^i)$, given as (29) and (30), respectively:

$$\mathbf{J}_{g_k^i(\mathbf{x}^i)} = \begin{pmatrix} (\Gamma_{k,c}^i I_{k,1}^{i,p} - \mathcal{P}_{k,d}^i(\tau_k^i)) |h_{k,cp}^i|^2 \\ (\Gamma_{k,c}^i I_{k,0}^{i,p} - (1 - \mathcal{P}_{k,d}^i(\tau_k^i))) |h_{k,cp}^i|^2 \\ \mathcal{P}_{k,d}^i(\tau_k^i)'(I_{k,1}^{i,p} - I_{k,0}^{i,p}) \end{pmatrix} \tag{29}$$

More specifically, if $\mathbf{x}^\star$ are the stationary solutions of game $\mathcal{G}(\mathbf{H}, \mathbf{G})$, and some CQ holds at $\mathbf{x}^\star$, the KKT conditions (24) can be reformulated to the equivalent form (31). The system of inequalities (31) defines a VI problem with variables $(\mathbf{x}, \boldsymbol{\alpha}, \boldsymbol{\beta})$, denoted as $VI(\mathbf{Q}, \boldsymbol{\Theta})$, where the vector function $\boldsymbol{\Theta}$

$$\mathbf{J}_{h^i(\mathbf{x}^i)} = \begin{pmatrix} \sum_{k=1}^{N}(1-\mathcal{P}(H_{0,k})\mathcal{P}^i_{k,fa}(\tau^i_k)-\mathcal{P}(H_{1,k})\mathcal{P}^i_{k,d}(\tau^i_k)) \\ \sum_{k=1}^{N}(\mathcal{P}(H_{0,k})\mathcal{P}^i_{k,fa}(\tau^i_k)+\mathcal{P}(H_{1,k})\mathcal{P}^i_{k,d}(\tau^i_k)) \\ \sum_{k=1}^{N}(\mathcal{P}(H_{1,k})\mathcal{P}^i_{k,d}(\tau^i_k)'+\mathcal{P}(H_{0,k})\mathcal{P}^i_{k,fa}(\tau^i_k)')(P^i_{k,1}-P^i_{k,0}) \end{pmatrix} \tag{30}$$

$$\begin{pmatrix} \mathbf{x}-\mathbf{x}^\star \\ \alpha_k-\alpha_k^\star \\ \beta-\beta^\star \end{pmatrix}^T \underbrace{\begin{pmatrix} \nabla_{\mathbf{x}^i} L(\mathbf{x}^{i,\star},\boldsymbol{\alpha}^{i,\star},\boldsymbol{\beta}^{i,\star}) \\ -g^i_k(\mathbf{x}^{i,\star}) \\ -h^i(\mathbf{x}^{i,\star}) \end{pmatrix}_{i=1}^{M}}_{\boldsymbol{\Theta}(\mathbf{x}^\star,\boldsymbol{\alpha}^\star,\boldsymbol{\beta}^\star)} \geq 0, \quad \forall (\mathbf{x}^i,\alpha^i_k,\beta^i) \in \underbrace{\prod_{i=1}^{M}\mathcal{X}^i \times \mathbb{R}^r_+}_{\mathbf{Q}} \tag{31}$$

and feasible set $\mathbf{Q}$ are defined in (31). This $VI(\mathbf{Q},\boldsymbol{\Theta})$ is an equivalent reformulation of the KKT conditions (24), where the convex constraints are embedded in the feasible set $\mathbf{Q}$, and r is the total number of multipliers $\boldsymbol{\alpha},\boldsymbol{\beta}$. The $VI(\mathbf{Q},\boldsymbol{\Theta})$ problem is to find a point $\mathbf{z}^\star = (\mathbf{x}^\star,\boldsymbol{\alpha}^\star,\boldsymbol{\beta}^\star) \in \mathbf{Q}$, such that $(\mathbf{z}-\mathbf{z}^\star)^T\boldsymbol{\Theta}(\mathbf{z}^\star) \geq 0$. In addition, if $(\mathbf{x}^\star,\boldsymbol{\alpha}^\star,\boldsymbol{\beta}^\star)$ is the solution of the $VI(\mathbf{Q},\boldsymbol{\Theta})$, there exists $\boldsymbol{\gamma}^\star$ such that $(\mathbf{x}^\star,\boldsymbol{\alpha}^\star,\boldsymbol{\beta}^\star,\boldsymbol{\gamma}^\star)$ is a solution of the game, $\boldsymbol{\gamma}^\star$ are the multipliers associated with the players' convex constraints $(b1),(b2)$ [28].

***Definition** 1:* A quasi-Nash equilibrium (QNE) of the game $\mathcal{G}(\mathbf{H},\mathbf{G})$ is defined and formed by the solution tuple $(\mathbf{x}^\star,\boldsymbol{\alpha}^\star,\boldsymbol{\beta}^\star)$ of the equivalent $VI(\mathbf{Q},\boldsymbol{\Theta})$, which is obtained under the first-order optimality conditions of each player's problems, while retaining the convex constraints in the defined set $\mathbf{Q}$. A QNE is said to be trivial, if $\mathbf{P}_0^\star,\mathbf{P}_1^\star = 0$ for all $i=1,...,M$ [27], [28].

B. The existence of the QNE

Note that a matrix $\mathbf{A}$ is copositive when $\mathbf{x}^T\mathbf{A}\mathbf{x} \geq 0$ for all $\mathbf{x} \geq 0$. $\mathbf{T}(\mathcal{X}^i;\mathbf{x}^i)$ denotes the tangent cone of the set $\mathcal{X}^i$ at $\mathbf{x}^i \in \mathcal{X}^i$ [34], i.e.,

$$\mathbf{T}(\mathcal{X}^i;\mathbf{x}^i) = \Bigg\{ \lim_{q\to\infty} \frac{\mathbf{x}^i_q-\mathbf{x}^i}{y_q} \mid \mathbf{x}^i_q \in \mathcal{X}^i,\, y_q \in \mathbb{R}_+ \text{ with } \lim_{q\to\infty}\mathbf{x}^i_q = \mathbf{x}^i,\, \lim_{q\to\infty} y_q = 0 \Bigg\}$$

***Theorem** 1:* The $VI(\mathbf{Q},\boldsymbol{\Theta})$ has a solution, and equivalently the game $\mathcal{G}(\mathbf{H},\mathbf{G})$ has a QNE, if the following conditions are satisfied [30]:

(A) Set $\mathcal{X}^i$ is convex, $i=1,...,M$.

(B) The function $\mathbf{F}(\mathbf{x}) = [-\nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i)]_{i=1}^M$ is continuously differentiable on its domain, and each $\mathbf{H}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are twice continuously differentiable on their domains.

(C) There exists a vector $\mathbf{x}^{ref} = [\mathbf{x}^{i,ref}]_{i=1}^M \in \mathcal{X}, \mathcal{X} = [\mathcal{X}^i]_{i=1}^M$, such that

- (C1) $\Psi^i(\mathbf{x}^{i,ref}) < 0$, where $\Psi^i(\mathbf{x}^{i,ref}) = \big(g^i_k(\mathbf{x}^{i,ref}),\ h^i(\mathbf{x}^{i,ref})\big)$.
- (C2) The Hessian matrix $\nabla^2_{\mathbf{x}^i} g^i_k(\mathbf{x}^i)$ is copositive on $\mathbf{T}(\mathcal{X}^i;\mathbf{x}^{i,ref})$ for $\mathbf{x}^i \in \mathcal{X}^i$.
- (C3) The Hessian matrix $\nabla^2_{\mathbf{x}^i} h^i(\mathbf{x}^i)$ is copositive on $\mathbf{T}(\mathcal{X}^i;\mathbf{x}^{i,ref})$ for $\mathbf{x}^i \in \mathcal{X}^i$.
- (C4) The set $\big\{\mathbf{x}^i \in \mathcal{X}^i | (\mathbf{x}^i-\mathbf{x}^{i,ref})\mathbf{F}^i(\mathbf{x}^i) \leq 0\big\}$ is bounded (possibly empty).

Proof: The non-convex problem $P2$ satisfies the hypotheses (A) and (B), and the proof for the hypotheses in (C1- C4) is given in Appendix A. ■

An interiority condition (C1) is needed for the non-convex constraints. Conditions (C2) and (C3) highlight the significance of distinguishing the non-convex constraints $\Psi^i(\mathbf{x}^{i,ref}) < 0$ from the convex constraints contained in each set $\mathcal{X}^i$. The condition (C4) is an assumption imposed for the existence of solutions of the $VI(\mathcal{X},\mathbf{F}(\mathbf{x}))$.
In order to show that the KKT conditions are valid necessary conditions for an optimal solution of $P2$, we need to verify that an appropriate CQ holds, as shown in [35]. In this paper, we use the linear independent constraint qualification (LICQ). If the gradients of the constraints are linearly independent at $\mathbf{x}^i$, we can prove that the LICQ holds at $\mathbf{x}^i$ [35].

***Lemma** 1:* The LICQ holds at every feasible solution of the problem $P2$.

Proof: Let the rank of $A^{m\times n}$ be denoted as $\mathcal{R}(A^{m\times n})$. Note that if $\mathcal{R}(A^{m\times n}) = \min(m,n)$, the matrix $A^{m\times n}$ is full rank and nonsingular. According to **Theorem 1**, problem $P2$ admits a solution $\mathbf{x}^{i,\star} = (\mathbf{P}_1^{i,\star},\mathbf{P}_0^{i,\star},\boldsymbol{\tau}^{i,\star})$, which is not trivial. Define the Jacobian matrix $\mathbf{J}_{\Psi^i(\mathbf{x}^{i,\star})} = \Big(\mathbf{J}_{g^i_k(\mathbf{x}^{i,\star})},\mathbf{J}_{h^i(\mathbf{x}^{i,\star})}\Big)$, where $\mathbf{J}_{g^i_k(\mathbf{x}^i)},\mathbf{J}_{h^i(\mathbf{x}^i)}$ are given by (29), (30), respectively. We can observe that in the first row of matrix $\mathbf{J}_{\Psi^i(\mathbf{x}^{i,\star})}$, the first item contains the variables $\mathbf{P}_1^i$ and $\boldsymbol{\tau}^i$, while the second item just contains the variable $\boldsymbol{\tau}^i$. Moreover, in the second row of matrix $\mathbf{J}_{\Psi^i(\mathbf{x}^{i,\star})}$, the variables in the first item are not equal to the ones in the second item. Hence, the first column $\mathbf{J}_{g^i_k(\mathbf{x}^{i,\star})}$ and the second column $\mathbf{J}_{h^i(\mathbf{x}^{i,\star})}$ are linear independent at $\mathbf{x}^{i,\star}$, if $|h^i_{k,cp}|^2 \neq 0$. The rank of $\mathbf{J}_{\Psi^i(\mathbf{x}^{i,\star})}$, defined as $\mathcal{R}(\mathbf{J}_{\Psi^i(\mathbf{x}^{i,\star})})$, is 2. Therefore, we can state that the Jacobian matrix $\mathbf{J}_{\Psi^i(\mathbf{x}^{i,\star})}$ is nonsingular for any given set of non-zero channel gains, and hence, the LICQ holds at every feasible solution of the problem $P2$. ■

Based on **Lemma 1**, we conclude that the KKT conditions are valid necessary conditions for an optimal solution of $P2$, namely, the achieved QNE coincides with the NE.

TABLE III
NOTATION OF PDIP OPTIMIZATION

Symbol	Value
$\mathbf{s}^i$	$(s^i_{k,0}, s^i_1, s^i_{k,2})^N_{k=1}$ Slack variables
$\mathbf{z}^i$	$(\mathbf{x}^i, \mathbf{s}^i)$
$\mathbf{v}^i$	(v^i_0, v^i_1, v^i_2) Barrier parameters
$\mathbf{u}^i$	$(\boldsymbol{\alpha}^i, \boldsymbol{\beta}^i, \boldsymbol{\gamma}^i)$
$\boldsymbol{\Lambda}^i$	$\text{Diag}(\mathbf{u}^i)$
$\mathcal{S}^i$	$\text{Diag}(\mathbf{s}^i)$
$M_{c^i}(\mathbf{z}^i)$	Merit function
$D_{M_{c^i}(\mathbf{z}^i;\mathbf{d}_{\mathbf{z}^i})}$	Directional derivative of $M_{c^i}(\mathbf{z}^i)$

V. PRIMAL-DUAL INTERIOR POINT OPTIMIZATION

The optimization problem $P1$ for CR i is non-convex with respect to $\mathbf{x}^i$, thus the optimal solution can not be obtained using conventional convex optimization techniques. In [1], we used the alternating direction optimization (ADO) algorithm for solving a similar non-convex problem. However, for non-convex problems, the ADO algorithm may not converge to the optimal solution, and hence, it can be considered as a local optimization algorithm [36].

The primal-dual interior point (PDIP) method is a powerful method for both convex and non-convex problems, which modifies the KKT conditions to ensure that the search direction is a descent direction for the merit function. In this paper, we analyze the iterative PDIP algorithm based on the IP algorithm from [37], [38], which combines a line search step and a trust region step. In addition, this PDIP algorithm requires no information exchange between CR users. We first compute the steps using line search whenever the conditions of these steps can be guaranteed, and turn to the trust region step otherwise. The trust region step, described in [38], starts by constructing a quadratic model of the Lagrangian function. The search direction is computed by minimizing the quadratic model, subject to the constraints and the trust region, which provides sufficient reduction in the merit function, and converges to a solution of $VI(\mathbf{Q}, \boldsymbol{\Theta})$, thus to a QNE of our game. The main symbols are given in Table III. The problem $P1$ can be reformulated as a sequence of the barrier problem $P3$:

$$\min_{\mathbf{z}^i} - f^i(\mathbf{x}^i) - v^i_0 \sum_{k=1}^{N} \ln s^i_{k,0} - v^i_1 \ln s^i_1 - v^i_2 \sum_{k=1}^{N} \ln s^i_{k,2}$$

$$\text{s.t. } g^i_k(\mathbf{x}^i) + s^i_{k,0} = 0 \tag{32}$$

$$h^i(\mathbf{x}^i) + s^i_1 = 0 \tag{33}$$

$$\tilde{g}^i_k(\mathbf{x}^i) + s^i_{k,2} = 0 \tag{34}$$

where $\tilde{g}^i_k(\mathbf{x}^i)$ denotes the convex constraints $(b1), (b2)$, $s^i_{k,0}, s^i_1, s^i_{k,2} > 0$ are vectors of slack variables, denoted as $\mathbf{s}^i = (s^i_{k,0}, s^i_1, s^i_{k,2})^N_{k=1}$. $v^i_0, v^i_1, v^i_2 > 0$ are the barrier parameters, denoted as $\mathbf{v}^i = (v^i_0, v^i_1, v^i_2)$. To simplify the problem, we denote $\mathbf{z}^i = (\mathbf{x}^i, \mathbf{s}^i)$, $\mathbf{u}^i = (\boldsymbol{\alpha}^i, \boldsymbol{\beta}^i, \boldsymbol{\gamma}^i)$, and $\varphi_{\mathbf{v}^i}(\mathbf{z}^i) = -f^i(\mathbf{x}^i) - v^i_0 \sum_{k=1}^{N} \ln s^i_{k,0} - v^i_1 \ln s^i_1 - v^i_2 \sum_{k=1}^{N} \ln s^i_{k,2}$. The Lagrangian function of the problem $P3$ is given by:

$$L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) = \varphi_{\mathbf{v}^i}(\mathbf{z}^i) + \sum\nolimits_{k=1}^{N} \alpha^i_k (g^i_k(\mathbf{x}^i) + s^i_{k,0}) + \beta^i (h^i(\mathbf{x}^i) + s^i_1) + \sum\nolimits_{k=1}^{N} \gamma^i_k (g^i_k(\tilde{\mathbf{x}}^i) + s^i_{k,2}) \tag{35}$$

Let $\boldsymbol{\Lambda}^i = \text{Diag}(\mathbf{u}^i)$, and $\mathcal{S}^i = \text{Diag}(\mathbf{s}^i)$, $\mathbf{e}$ is the all-ones column vector. The first order optimality conditions of the problem $P3$ can be written as:

$$\nabla_{\mathbf{z}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) = \begin{pmatrix} \nabla_{\mathbf{x}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) \\ \mathcal{S}^i \boldsymbol{\Lambda}^i \mathbf{e} - \mathbf{v}^i \mathbf{e} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{36}$$

where $\nabla_{\mathbf{x}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i)$ is given by:

$$\nabla_{\mathbf{x}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) = - \nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i) + \sum\nolimits_{k=1}^{N} \alpha^i_k \mathbf{J}_{g^i_k(\mathbf{x}^i)} + \beta^i \mathbf{J}_{h^i(\mathbf{x}^i)} + \sum\nolimits_{k=1}^{N} \gamma^i_k \mathbf{J}_{\tilde{g}^i_k(\mathbf{x}^i)} \tag{37}$$

$\nabla_{\mathbf{x}^i} f^i(\mathbf{x}^i), \mathbf{J}_{g^i_k(\mathbf{x}^i)}, \mathbf{J}_{h^i(\mathbf{x}^i)}$ are given by (25)-(27), (29) and (30), respectively. The $\mathbf{J}_{\tilde{g}^i_k(\mathbf{x}^i)}$ is the Jacobian matrix of the convex constraints $\tilde{g}^i_k(\mathbf{x}^i)$. Applying Newton's method to problem $P3$, we obtain the following primal-dual system:

$$\begin{pmatrix} \mathbf{W}(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) & \mathbf{J}(\mathbf{x}^i) \\ \mathbf{J}^T(\mathbf{x}^i) & 0 \end{pmatrix} \begin{pmatrix} \mathbf{d}_{\mathbf{z}^i} \\ \mathbf{d}_{\mathbf{u}^i} \end{pmatrix} = \begin{pmatrix} \nabla_{\mathbf{z}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) \\ \mathbf{B}(\mathbf{z}^i) \end{pmatrix} \tag{38}$$

$\mathbf{B}(\mathbf{z}^i)$ is defined as:

$$\mathbf{B}(\mathbf{z}^i) = \begin{pmatrix} g^i_k(\mathbf{x}^i) + s^i_{k,0} \\ h^i(\mathbf{x}^i) + s^i_1 \\ \tilde{g}^i_k(\mathbf{x}^i) + s^i_{k,2} \end{pmatrix}^N_{k=1} \tag{39}$$

and $\mathbf{W}(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i)$ is defined as:

$$\mathbf{W}(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) = \begin{pmatrix} \nabla^2_{\mathbf{x}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i) & 0 \\ 0 & (\mathcal{S}^i)^{-1} \boldsymbol{\Lambda}^i \end{pmatrix} \tag{40}$$

where $\nabla^2_{\mathbf{x}^i} L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i)$ is the Hessian matrix of $L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i)$, and $\mathbf{J}(\mathbf{x}^i)$ is given by:

$$\mathbf{J}(\mathbf{x}^i) = \begin{pmatrix} \mathbf{J}_{g^i_k(\mathbf{x}^i)} & \mathbf{J}_{h^i(\mathbf{x}^i)} & \mathbf{J}_{\tilde{g}^i_k(\mathbf{x}^i)} & \mathbf{I} \end{pmatrix}^N_{k=1} \tag{41}$$

We define the search directions $\mathbf{d}_{\mathbf{z}^i}$ and $\mathbf{d}_{\mathbf{u}^i}$ as:

$$\mathbf{d}^T_{\mathbf{z}^i} = \begin{pmatrix} d_{P^i_{k,0}}, & d_{P^i_{k,1}}, & d_{\tau^i_k}, & d_{s^i_{k,0}}, & d_{s^i_1}, & d_{s^i_{k,2}} \end{pmatrix}^N_{k=1}$$

$$\mathbf{d}^T_{\mathbf{u}^i} = \begin{pmatrix} d_{\alpha^i_k}, & d_{\beta^i}, & d_{\gamma^i_k} \end{pmatrix}^N_{k=1}$$

The objective function component and the component comprising constraints of the problem $P3$ are used as the merit function for the PDIP algorithm, which can be defined by:

$$M_{c^i}(\mathbf{z}^i) = \varphi_{\mathbf{v}^i}(\mathbf{z}^i) + c^i \|\mathbf{B}(\mathbf{z}^i)\| \tag{42}$$

where $c^i > 0$ is the penalty parameter, which is updated at each iteration so that the search direction $\mathbf{d}_{\mathbf{z}^i}$ is a descent direction for $M_{c^i}(\mathbf{z}^i)$. The iterations are given by:

$$\mathbf{z}^i(p+1) = \mathbf{z}^i(p) + \boldsymbol{\rho}^i_{\mathbf{z}^i} \mathbf{d}_{\mathbf{z}^i}(p) \tag{43}$$

$$\mathbf{u}^i(p+1) = \mathbf{u}^i(p) + \boldsymbol{\rho}^i_{\mathbf{u}^i} \mathbf{d}_{\mathbf{u}^i}(p) \tag{44}$$

where p is the number of the inner iteration loop, $\boldsymbol{\rho}^i_{\mathbf{z}^i}$ and $\boldsymbol{\rho}^i_{\mathbf{u}^i}$ are the step-lengths. We then perform a backtracking line search that computes the step-lengths which provide a sufficient decrease in the merit function. The step-lengths $\boldsymbol{\rho}^i_{\mathbf{z}^i}, \boldsymbol{\rho}^i_{\mathbf{u}^i} \in (0, 1]$ are given by:

$$\boldsymbol{\rho}^i_{\mathbf{z}^i} = \{\mathbf{s}^i + \boldsymbol{\rho}^i_{\mathbf{z}^i} \mathbf{d}_{\mathbf{s}^i} \geq \xi_0 \mathbf{s}^i\} \tag{45}$$

$$\boldsymbol{\rho}^i_{\mathbf{u}^i} = \{\mathbf{u}^i + \boldsymbol{\rho}^i_{\mathbf{u}^i} \mathbf{d}_{\mathbf{u}^i} \geq \xi_0 \mathbf{u}^i\} \tag{46}$$

where $\xi_0 \in (0,1]$ is a constant. Moreover, the directional derivative of $M_{c^i}(\mathbf{z}^i)$ is given by:

$$D_{M_{c^i}(\mathbf{z}^i;\mathbf{d}_{\mathbf{z}^i})} = \nabla\varphi_{\mathbf{v}^i}(\mathbf{z}^i)\mathbf{d}_{\mathbf{z}^i} - c^i\|\mathbf{B}(\mathbf{z}^i)\| \tag{47}$$

Expressions (38)-(45) provide the basis for the line search steps in the PDIP algorithm. However, due to the non-convexity of the problem $P3$, the line search iterations may converge to non-stationary points. If the step-lengths $\boldsymbol{\rho}^i_{\mathbf{z}^i}, \boldsymbol{\rho}^i_{\mathbf{u}^i}$ converge to zero, we turn to the trust region iterations, which provide a sufficient reduction in the chosen merit function for both feasibility and optimality at every iteration and thus, guarantee progress towards stationary [38].

The trust region step treats convex and non-convex problems uniformly, and allows the direct use of the second derivative information. In addition to preserving the global convergence properties of the trust region step, the size of a trust region radius Υ^i affects the backtracking line search iterations. Note that if a trust region iteration is rejected, the following iterations are still computed by the trust region step until a successful step is obtained. In the trust region step, a step $\mathbf{d}$ is acceptable if the ratio of actual reduction (ared($\mathbf{d}$)) to predicted reduction (pred($\mathbf{d}$)) of the merit function is greater than a given constant $\eta > 0$, denoted as (48), where $\mathbf{W}$ is defined in (40). We outline the iterative PDIP algorithm in **Algorithm** 1, where N^i_e is the number of negative eigenvalues of the matrix in (38), and N_b is the maximum number of backtracking search steps. For our problem, if $N^i_e > 4N$, then $\mathbf{d}_{\mathbf{z}^i}$ can not be guaranteed to be the descent direction [39]. In this case, we turn to the trust region steps. We choose $\eta = 10^{-8}$, $\varepsilon = 10^{-6}$, and $N_b = 4$. The resulting algorithm is ensured to have global convergence, thus achieving a QNE of the $VI(\mathbf{Q}, \boldsymbol{\Theta})$. For more details of the trust region iterations and the global convergence analysis, refer to [37], [38].

Complexity analysis. The complexity of the iterative PDIP algorithm is dominated by the procedure of line search iteration steps and trust region iteration steps, as well as the size of the CRN. Generally, for the inner loop, the time complexity of line search is based on the Newton iteration, which requires at most $O((2N+M)^3)$ computations. For the ε-accurate iteration, the computation of Newton iterations reduce to $O(\ln(\frac{1}{\varepsilon})\sqrt{2N+M})$ [40], and according to [41], the complexity for the logarithmic barrier function is the best one given by $O(\sqrt{2N+M})$. For our problem, the maximum number of backtracking search steps is given by N_b, thus the time complexity of the line search is $O(\sqrt{2N+M}) \sim O(N_b\sqrt{2N+M})$. In addition, the trust region iterations step is based on the sequential quadratic programming techniques [42], [43], and the worst-case complexity of reaching a scaled stationary point is $O(2N+M+\sqrt{2N+M})$ [44]. The outer loop for a CRN with M CR users is a linear problem with the accuracy ε, thus the total complexity of the PDIP algorithm is given by $O_{PDIP} = O\Bigg(\ln(\frac{1}{\varepsilon})M\sqrt{2N+M}\Bigg) \sim O\Bigg(\ln(\frac{1}{\varepsilon})M((N_b+1)\sqrt{2N+M}+2N+M)\Bigg)$. Notice that here we did not consider the time complexity of the convergence of the consensus algorithm in the cooperative sensing step.

Algorithm 1 Primal-Dual Interior Point Optimization

Initialize $\mathbf{z}^i(0) = (\mathbf{x}^i(0), \mathbf{s}^i(0))$. Compute initial values for the multipliers $\mathbf{u}^i(0) = (\boldsymbol{\alpha}^i(0), \boldsymbol{\beta}^i(0), \boldsymbol{\gamma}^i(0))$, set the trust-region radius $\Upsilon^i(0) > 0$ and the barrier parameter $\mathbf{v}^i(0) > 0$.
repeat
  for i =1: M
    repeat
      repeat
      Compute the number N^i_e from (38), set LS = 0
        if $N^i_e \leq 4N$
        Calculate the search direction $\mathbf{d}(p) = (\mathbf{d}_{\mathbf{z}^i}(p), \mathbf{d}_{\mathbf{u}^i}(p))$ from (38). Compute $\boldsymbol{\rho}^i_{\mathbf{z}^i}$, $\boldsymbol{\rho}^i_{\mathbf{u}^i}$
          if $\min\{\boldsymbol{\rho}^i_{\mathbf{z}^i}, \boldsymbol{\rho}^i_{\mathbf{u}^i}\} > \varepsilon$
          Set $j = 0, \boldsymbol{\rho}^i_T = 1$
            repeat
              if $M_{c^i}(\mathbf{z}^i(p)+\boldsymbol{\rho}^i_T\boldsymbol{\rho}^i_{\mathbf{z}^i}\mathbf{d}_{\mathbf{z}^i}(p)) \leq M_{c^i}(\mathbf{z}^i(p)) + \eta\boldsymbol{\rho}^i_T\boldsymbol{\rho}^i_{\mathbf{z}^i}D_{M_{c^i}(\mathbf{z}^i;\mathbf{d}_{\mathbf{z}^i})}$
              Update $\boldsymbol{\rho}^i_{\mathbf{z}^i} = \boldsymbol{\rho}^i_T\boldsymbol{\rho}^i_{\mathbf{z}^i}$, $\boldsymbol{\rho}^i_{\mathbf{u}^i} = \boldsymbol{\rho}^i_T\boldsymbol{\rho}^i_{\mathbf{u}^i}$
              Update $\mathbf{z}^i(p+1), \mathbf{u}^i(p+1)$ using (43). Update $\Upsilon^i(p+1)$. Set LS = 1
              else Update $j = j+1$, choose a smaller value of $\boldsymbol{\rho}^i_T$
              endif
            until $j > N_b$ or $\boldsymbol{\rho}^i_T < \varepsilon$ Or LS == 1
          endif
        endif
        if LS == 0
        Compute the step $\mathbf{d}(p) = (\mathbf{d}_{\mathbf{z}^i}(p), \mathbf{d}_{\mathbf{u}^i}(p))$
        Compute Lagrange multiplier $\mathbf{u}^i(p+1)$. Update the penalty parameter c^i
          if ared$(\mathbf{d}) \geq \eta$pred$(\mathbf{d})$
          Set $\mathbf{z}^i(p+1) = \mathbf{z}^i(p) + \mathbf{d}_{\mathbf{z}^i}(p)$. Enlarge the trust region radius $\Upsilon^i(p+1)$
          else Set $\mathbf{z}^i(p+1) = \mathbf{z}^i(p)$. Shrink the trust region $\Upsilon^i(p+1)$
          endif Set $\mathbf{v}^i(p+1) = \mathbf{v}^i(p)$, $p = p+1$
      until $\|\nabla_{\mathbf{x}^i}L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i)\|_\infty \leq \varepsilon$ and $\|\mathcal{S}^i\mathbf{e}\boldsymbol{\Lambda}^i - \mathbf{v}^i\mathbf{e}\|_\infty \leq \varepsilon$
      Reset the barrier parameters, so that $\mathbf{v}^i(p+1) < \mathbf{v}^i(p)$
    until $\|\nabla_{\mathbf{x}^i}L(\mathbf{z}^i, \mathbf{u}^i; \mathbf{v}^i)\|_\infty \leq \varepsilon$ and $\|\mathcal{S}^i\boldsymbol{\Lambda}^i\|_\infty \leq \varepsilon$
    Update $\mathbf{x}^i(p_0) = \mathbf{x}^i(p)$, where p_0 is the number of the outer loop.
  endfor
until $\|\mathbf{x}^i(p_0) - \mathbf{x}^i(p_0-1)\| \leq \varepsilon$

VI. Simulation Results

A. Scenario Description

We consider a CRN with $M = 3$ CR Tx-Rx pairs and $N = 2$ PU channels. All PUs and CR users are randomly placed in a 50 meter $\times$ 50 meter square. The radio environment map is shown in Fig.2, where the color-bar shows the received power from PUs in Watt. We use the channel model from the 3GPP Indoor scenario for LTE [45]. The distance-dependent path loss is given by $PL_{dB} = 7 + 56\log_{10}(d)$; $d = d_{ji}/d_{ii}$ (m) is the relative distance between CR-Tx j and CR-Rx i,

$$\eta < \frac{\text{ared}(\mathbf{d})}{\text{pred}(\mathbf{d})} = \frac{M_{c^i}(\mathbf{z}^i) - M_{c^i}(\mathbf{z}^i + \mathbf{d}_{\mathbf{z}^i})}{-\nabla\varphi_{\mathbf{v}^i}(\mathbf{z}^i)\mathbf{d}_{\mathbf{z}^i} - \frac{1}{2}\mathbf{d}_{\mathbf{z}^i}^T W \mathbf{d}_{\mathbf{z}^i} + c^i(\|\mathbf{B}(\mathbf{z}^i)\| - \|\mathbf{B}(\mathbf{z}^i) + J(\mathbf{x}^i)\mathbf{d}_{\mathbf{z}^i}\|)} \tag{48}$$

TABLE IV
SIMULATION PARAMETER

Symbol	Value
Sensing time t	1ms
Sampling frequency, f_s	$2MHz$
Probability of channel k idle, $\mathcal{P}(H_{0,k})$	$0.1, 0.5$
Probability of channel k occupied, $\mathcal{P}(H_{1,k})$	$0.9, 0.5$
Transmit power budget of CR i, $P^i_{\max}$	$0 \sim 10W$
Transmit power of PU in channel k, $\lvert S_k \rvert^2$	$10W$
Rate-loss gap of channel k, Γ_k	0.1%,0.3%,1%

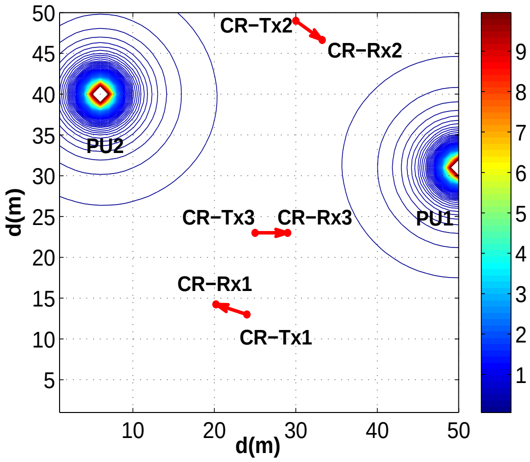

Fig. 2. Network topology: Location of two PU and three CR Tx-Rx pairs.

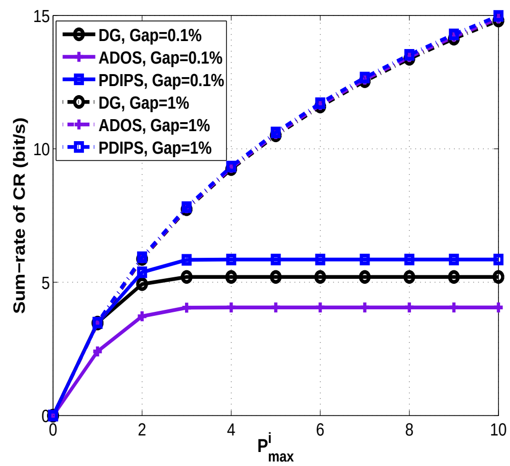

Fig. 3. Sum-rate achieved at the QNE versus $P^i_{\max}$; Comparison between ADO algorithm and DG.

where d_{ii} and d_{ji} are the distances between CR-Tx i and CR-Rx i, CR-Tx j and CR-Rx i, respectively. A lognormal shadowing variable with variance 10 dBs is considered here, and $(\sigma^i_{k,n})^2 = 1$. Assume that the sensing environment is stable in the optimization process, and the local channel state information, i.e., the channel gain between a CR-Tx and its target Rx and each PU, is known by each CR-Tx. The main simulation parameters are given in Table IV.

B. Simulation Results

In this section, we first compare the performance of the proposed game, in terms of the sum-rate achieved at the QNE for one CR user by the PDIP algorithm, with those achieved by the ADO algorithm [1] and the deterministic game (DG) proposed in [25]. Then, we investigate the influence of the activity of the PU and compare the sum-rate achieved by different constraints, respectively. Finally, we show the actual rate-loss of the PUs under constraints (15) and (17).

In Fig. 3, we compare the sum-rate achieved at the QNE by the PDIP algorithm with those achieved by the ADO algorithm and the DG. For the ADO algorithm, the first step is to maximize the sum-rate of each CR i based on an initial detection threshold, and then optimize the threshold based on the optimal power obtained in the first step. The sensing information is not considered as a part of optimization for the DG. Regarding the constraint inequalities given in (17), (18), it can be seen that the optimization problem works in two possible regimes:

(a) Power budget limited regime (PLR), where the transmit power is bounded by the total power budget constraint (18), leading to the worst case interference condition. In this case, each CR-Tx i allocates all the available power budget and causes the maximum interference to other CR-Rxs [6], and the achievable rate is determined by the total power budget.
(b) Rate-loss limited regime (RLR), which implies that the transmit power is bounded by the rate-loss constraint (17). Increasing the total power budget can not improve the performance of the CR users, and the interference to the PUs reaches the upper bound.

The results shows that when the CR users work in PLR, when $\Gamma_k = 1\%$, the performances of these three algorithms are almost the same, while the proposed game with joint optimization of the sensing information and transmit power by PDIP algorithm yields a considerable performance improvement in RLR, when $\Gamma_k = 0.1\%$, with respect to the ADO algorithm and the disjoint case of the DG. In fact, the DG can be considered as the perfect sensing information case (i.e. $\mathcal{P}^i_{k,fa} = 0$ and $\mathcal{P}^i_{k,d} = 1$) with a deterministic interference constraint. Specifically, in RLR, a higher transmit power is allowed due to the accurate sensing information in the proposed game compared to the DG with a deterministic interference constraint, thus the performance can be improved. In addition, when $\Gamma_k = 0.1\%$, the sum-rate of CR users does not change after $P^i_{\max} > 1W$, indicating that the transmit power changes from PLR to RLR. Fig. 4 presents the sum-rate achieved at the QNE versus the power budget $P^i_{\max}$ for different average

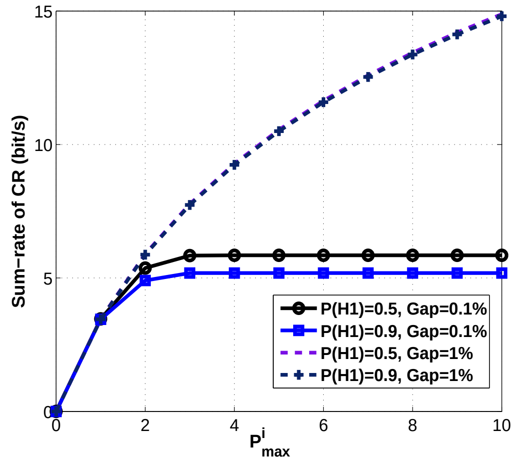

Fig. 4. Sum-rate achieved at the QNE versus $P^i_{\max}$; Comparison between $\mathcal{P}(H_{k,1}) = 0.5$ and $\mathcal{P}(H_{k,1}) = 0.9$.

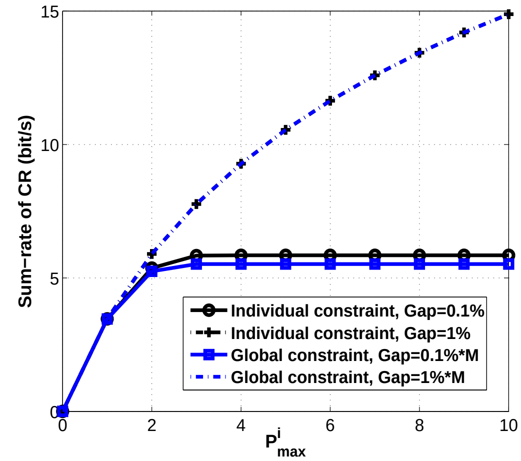

Fig. 5. Sum-rate achieved at the QNE versus $P^i_{\max}$; Comparison between Global constraint and Individual constraint.

fractions of the PU's activity, $\mathcal{P}(H_{1,k}) = 0.5, 0.9$, which are directly related to the traffic load of the PU. It can be observed that in RLR, when $\Gamma_k = 0.1\%$, the traffic load of the PUs affects the sum-rate of the CR users. The CR users suffer a decrease in sum-rate when the traffic load of the PU increases from 0.5 to 0.9. In other words, when there is more activity of the PU, there is less chance for the CR users to use the channel. Additionally, in PLR, the performance of the CR users is not sensitive to the traffic load of the PU.

In Fig. 5, we compare the performance achieved by the global constraint with the individual constraint (17), respectively. In order to have the same total interference to the PU, we use a rate-loss gap $\Gamma_{k,g} = \Gamma_k \times M$ for the global constraint. Based on the individual constraint (17), the global constraint can be written as $(1-\Gamma_{k,g})I^{i,pt}_{k,1} I^{i,pt}_{k,0} - \mathcal{P}^i_{k,d} I^{i,pt}_{k,0} - (1-\mathcal{P}^i_{k,d}) I^{i,pt}_{k,1} \leq 0$, where $I^{i,pt}_{k,0}, I^{i,pt}_{k,1}$ stand for the total interference from all the CR users. It is rather interesting to notice that when the rate-loss constraint is active, the performance of the CR users under the individual constraint is better than those achieved by the global constraint. However, this is due to the unfairness among the CR users in the global constraint. Each iteration of the game follows a sequential order, indicating that the CR users having the priority to choose their action can have the preference to maximize their own benefit in the global constraint case, and the CR users at the bottom of the iteration loop have to be switched off in RLR. These inherently unfairness for the global constraint leads to a lower utilization of the channel, yielding a worst performance of the CR users. Actually, the global constraint can result in a better performance than the individual constraint by pricing mechanism, which uses a penalty in the objective function and encourages the CR users to work in a cooperative manner to achieve a higher social welfare [28], [29], [46].

Finally, in Fig. 6, we evaluate the interference experienced by the PU under constraint (15) and the modified constraint (17). The rate-loss gap is defined as $(R^i_{k,\max} - R^i_k)/R^i_{k,\max}$, and $R^i_{k,\max}$, R^i_k are given by (11), (12), respectively. It can be observed that in RLR, the constraint (15) imposes a less

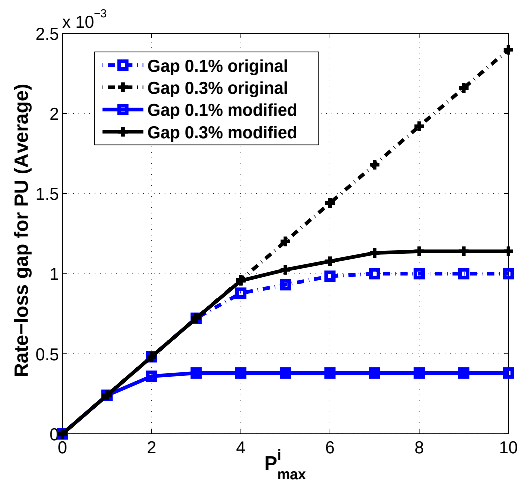

Fig. 6. Average-rate gap for PU achieved at the QNE versus $P^i_{\max}$; Comparison between constraints (15) and (17).

strict condition on the transmit power of the CR users than the one imposed by the modified constraint (17). This leads to a higher interference and a larger rate-loss gap experienced by the PUs, and to an increase of the sum-rate of the CR users. In other words, the modified constraint (17) can be seen as the constraint (15) with a smaller rate-loss gap.

VII. Conclusions

In this paper, we considered a sensing-based spectrum sharing scenario, where the overall objective was to maximize the sum-rate of each cognitive radio user by optimizing jointly both the detection operation and the power allocation. In order to deal with the non-convexity of the game, we used a relaxed equilibria concept, the quasi-Nash equilibrium (QNE). We presented the sufficient conditions for the existence of a QNE based on variational inequality theory, and proved that the linear independent constraint qualification held at every feasible solution of the proposed game, thus the achieved QNE coincided with the NE. Finally, a distributed iterative

$$\nabla^2_{\mathbf{x}^i} g^i_k(\mathbf{x}^i) = \begin{pmatrix} 0 & \Gamma^i_{k,c}|h^i_{k,cp}|^4 & -\mathcal{P}^i_{k,d}(\tau^i_k)'|h^i_{k,cp}|^2 \\ \Gamma^i_{k,c}|h^i_{k,cp}|^4 & 0 & \mathcal{P}^i_{k,d}(\tau^i_k)'|h^i_{k,cp}|^2 \\ -\mathcal{P}^i_{k,d}(\tau^i_k)'|h^i_{k,cp}|^2 & \mathcal{P}^i_{k,d}(\tau^i_k)'|h^i_{k,cp}|^2 & \mathcal{P}^i_{k,d}(\tau^i_k)''(I^{i,p}_{k,1} - I^{i,p}_{k,0}) \end{pmatrix} \tag{49}$$

$$\begin{aligned}(\mathbf{x}^i - \mathbf{x}^{i,ref})^T \nabla^2_{\mathbf{x}^i} g^i_{k,c}(\mathbf{x}^i)(\mathbf{x}^i - \mathbf{x}^{i,ref}) &= (P^i_{k,0} - P^{i,ref}_{k,0})^2(1 - \mathcal{P}^i_{k,d}(\tau^i_k))U^{i,p}_{k,0} + (P^i_{k,1} - P^{i,ref}_{k,1})^2\mathcal{P}^i_{k,d}(\tau^i_k)U^{i,p}_{k,1} \\ &+ 2(P^i_{k,0} - P^{i,ref}_{k,0})(\tau^i_k - \tau^{i,ref}_k)\mathcal{P}^i_{k,d}(\tau^i_k)'|h^i_{k,cp}|^2((I^{i,p}_{k,0} + |S^i_k|^2)^{-1} - (I^{i,p}_{k,0})^{-1}) \\ &+ 2(P^i_{k,1} - P^{i,ref}_{k,1})(\tau^i_k - \tau^{i,ref}_k)\mathcal{P}^i_{k,d}(\tau^i_k)'|h^i_{k,cp}|^2((I^{i,p}_{k,1})^{-1} - (I^{i,p}_{k,1} + |S^i_k|^2)^{-1}) \\ &+ (\tau^i_k - \tau^{i,ref}_k)^2\mathcal{P}^i_{k,d}(\tau^i_k)'' \log_2((1 + |S^i_k|^2/I^{i,p}_{k,1})/(1 + |S^i_k|^2/I^{i,p}_{k,0}))\end{aligned} \tag{50}$$

primal-dual interior point algorithm was stated and shown to converge to a QNE of the proposed game. Simulation results showed that the iterative primal-dual interior point algorithm yielded a considerable performance improvement with respect to the alternating direction optimization algorithm and the deterministic game.

Appendix A
Proof of the hypotheses in Theorem 1

Due to lack of space, only the sketch is provided. The Hessian matrix $\nabla^2_{\mathbf{x}^i} g^i_k(\mathbf{x}^i)$ is given by (49), where $\Gamma^i_{k,c} = (1-\Gamma_k)/(\sigma^i_{k,n})^2$. In order to check that conditions (C1), (C2) and (C3) are satisfied, we assume that $P^{i,ref}_{k,0} = 0$, $P^{i,ref}_{k,1} = 0$ and $\tau^{i,ref}_k = \tau^i_{k,min}$, where $\tau^i_k \in [\tau^i_{k,min}, \tau^i_{k,max}]$. It follows that $\mathbf{x}^{i,ref} = [P^{i,ref}_{k,0}, P^{i,ref}_{k,1}, \tau^{i,ref}_k]^N_{k=1}$, and we have:

$$\begin{aligned}&(\mathbf{x}^i - \mathbf{x}^{i,ref})^T \nabla^2_{\mathbf{x}^i} g^i_k(\mathbf{x}^i)(\mathbf{x}^{ref} - \mathbf{x}^{i,ref}) \\ &= 2\Gamma^i_{k,c}P^i_{k,0}P^i_{k,1}|h^i_{k,cp}|^4 + (\tau^i_k - \tau^{i,ref}_k)^2\mathcal{P}^i_{k,d}(\tau^i_k)''(I^{i,p}_{k,1} - I^p_{k,0}) \\ &+ 2(\tau^i_k - \tau^{i,ref}_k)\mathcal{P}^i_{k,d}(\tau^i_k)'(P^i_{k,1} - P^i_{k,0})|h^i_{k,cp}|^2\end{aligned}$$

Notice that $P^i_{k,1} < P^i_{k,0}$, $I^{i,p}_{k,1} < I^p_{k,0}$, $\mathcal{P}^i_{k,d}(\tau^i_k)' < 0$ and $\mathcal{P}^i_{k,d}(\tau^i_k)'' < 0$. All the terms are positive, thus the Hessian matrix of $\nabla^2_{\mathbf{x}^i} g^i_k(\mathbf{x}^i)$ is copositive. Similarly, we can show that the Hessian matrix of function $h^i(\mathbf{x}^i)$ is copositive. Thus, conditions (C1), (C2) and (C3) are satisfied. For condition (C4), we need to show that the player's variables $\mathbf{x} = (\mathbf{P}_0, \mathbf{P}_1, \boldsymbol{\tau})$ are bounded. For every CR i, we have $0 \leq P^i_{k,0}$ and $0 \leq P^i_{k,1}$, and from power budget constraint (18) we can get:

$$P^i_{k,0} \leq \frac{P^i_{\max}}{(1 - \mathcal{P}(H_{0,k})\mathcal{P}^i_{k,fa}(\tau^i_k) - \mathcal{P}(H_{1,k})\mathcal{P}^i_{k,d}(\tau^i_k))} \leq \frac{P^i_{\max}}{A^i_{k,0}}$$

$$P^i_{k,1} \leq \frac{P^i_{\max}}{(\mathcal{P}(H_{0,k})\mathcal{P}^i_{k,fa}(\tau^i_k) + \mathcal{P}(H_{1,k})\mathcal{P}^i_{k,d}(\tau^i_k))} \leq \frac{P^i_{\max}}{A^i_{k,1}}$$

where $A^i_{k,0} = 1 - \frac{1}{2}\mathcal{P}(H_{0,k}) - \mathcal{P}(H_{1,k})\mathcal{Q}\left(\frac{\mu^i_{k,0} - \mu^i_{k,1}}{\sigma^i_{k,1}}\right)$, $A^i_{k,1} = \frac{1}{2}\mathcal{P}(H_{1,k}) + \mathcal{P}(H_{0,k})\mathcal{Q}\left(\frac{\mu^i_{k,1} - \mu^i_{k,0}}{\sigma^i_{k,0}}\right)$. In addition, $\boldsymbol{\tau}$ is bounded by the constraint (19), and we can conclude that the condition (C4) is also satisfied. Therefore, the $VI(\mathbf{Q}, \boldsymbol{\Theta})$ has a solution, and the game $\mathcal{G}(\mathbf{H}, \mathbf{G})$ has a QNE. Moreover, every QNE is not trivial, a trivial QNE can not satisfy (31).

Constraint (15) v.s. Constraint (17): For Constraint (15), denoted as $g^i_{k,c}(\mathbf{x}^i)$, we have (50), where $U^{i,p}_{k,0} = |h^i_{k,cp}|^4((I^{i,p}_{k,0} + |S^i_k|^2)^{-2} - (I^{i,p}_{k,0})^{-2})$, $U^{i,p}_{k,1} = |h^i_{k,cp}|^4((I^{i,p}_{k,1} + |S^i_k|^2)^{-2} - (I^{i,p}_{k,1})^{-2})$. The first and the second term on the right side are negative, the fifth term is positive, the sum of the third and the forth term can be proved to be positive. Hence, assuming $U^{i,p}_{k,0} > U^{i,p}_{k,1}$, the $\nabla^2_{\mathbf{x}^i} g^i_{k,c}(\mathbf{x}^i)$ is copositive if the following inequality is satisfied:

$$\begin{aligned}&(\tau^i_k - \tau^{i,ref}_k)^2\mathcal{P}^i_{k,d}(\tau^i_k)'' \log_2(1 + |S^i_k|^2/I^{i,p}_{k,1})/(1 + |S^i_k|^2/I^{i,p}_{k,0})) \\ &> ((\mathcal{P}^i_{k,d}(\tau^i_k) - 1)(P^i_{k,0})^2 - \mathcal{P}^i_{k,d}(\tau^i_k)(P^i_{k,1})^2)U^{i,p}_{k,0}\end{aligned} \tag{51}$$

However, this condition depends on the values of the system parameters as well as the action of the CR i, which is uncertain. In order to simplify the analysis, we use constraint (17) instead of constraint (15), which is more suitable for a general network, and offers a better protection for PU, as shown in the simulation results.

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Xiaoge Huang received her B.Sc. and M.Sc. degrees in telecommunication engineering (both with first honors) from the Chongqing University of Posts and Telecommunications (CQUPT), China, in 2005 and 2008, respectively. She is currently working toward her Ph.D. degree in the Group of Information and Communication Systems (GSIC), Institute of Robotics and Information & Communication Technologies (IRTIC) at the University of Valencia, Spain. Her research interests include convex optimization, centralized and decentralized power allocation strategies, game theory, cognitive radio networks, and multiuser MIMO systems.

Baltasar Beferull-Lozano (M'01, SM'08) received his M.Sc. in Physics from Universidad de Valencia, Spain, in 1995 (First in Class Honors) and the M.Sc. and Ph.D. degrees in Electrical Engineering from University of Southern California, Los Angeles, in 1999 and 2002, respectively. His PhD work was supported by a National Graduate Doctoral Fellowship from the Ministry of Education of Spain.

From January 1996 to August 1997, he was a Research (Fellow) Assistant at the Department of Electronics and Computer Science, Universidad de Valencia, and from September 1997 to September 2002, he was a Research (Fellow) Assistant in the Department of Electrical Engineering, the Integrated Media Systems Center (NSF Engineering Research Center) and the Signal and Image Processing Institute, at the University of Southern California. He has also worked for AT&T Shannon Laboratories (formerly AT&T Bell Laboratories), Information Sciences Center, Florham Park, NJ. From October 2002 to June 2005, he was a Research Associate in the Department of Communication Systems at the Swiss Federal Institute of Technology-EPFL, Lausanne, Switzerland, and a Senior Researcher within the Swiss National Competence Center in Research on Mobile Information and Communication Systems (Swiss NSF Research Center). From July 2005 to November 2005, he was a Visiting Senior Researcher at Universidad de Valencia and Universidad Politécnica de Valencia. In December 2005, he joined Universidad de Valencia, where he is now an Associate Professor and the Head of the Group of Information and Communication Systems. His research interests are in the general areas of signal and image processing, distributed signal processing and communications for sensor networks, information theory and communication theory. He has served as a member of the Technical Program Committees for several ACM & IEEE International Conferences and holds a Telefonica Chair.

At University of Southern California, Dr. Beferull-Lozano received several awards including the Best PhD Thesis paper Award in April 2002 and the Outstanding Academic Achievement Award both in April 1999 and April 2002. He also received a Best Paper Award at the IEEE International Conference on Distributed Computing in Sensor Systems (DCOSS) 2012.

Carmen Botella (S'03, M'09) received her M.Sc. and Ph.D. degrees in Telecommunications engineering from Universidad Politécnica de Valencia, Spain, in 2003 and 2008, respectively. In 2009 and 2010, she was a postdoctoral researcher in the Communications Systems and Information Theory group, Chalmers University of Technology, Sweden. In 2011, she joins the Group of Information and Communication Systems, University of Valencia, as an Assistant Professor. Her research interests include the general areas of coordination and cooperation in wireless systems.