Low-Complexity Near-Optimal Decoding for Analog Joint Source Channel Coding Using Space-Filling Curves

O. Fresnedo, *Member, IEEE,* F. J. Vazquez-Araujo, *Member, IEEE,* L. Castedo, *Member, IEEE,* and J. Garcia-Frias, *Senior Member, IEEE*

***Abstract*—Analog Joint Source-Channel Coding (JSCC) is a communication strategy that does not follow the separation principle of conventional digital systems but approaches the optimal distortion-cost tradeoff over AWGN channels. Conventional Maximum Likelihood (ML) analog JSCC decoding schemes suffer performance degradation at low Channel Signal to Noise Ratio (CSNR) values, while Minimum Mean Square Error (MMSE) decoding presents high complexity. In this letter we propose an alternative two step decoding approach which achieves the near-optimal performance of MMSE decoding at all CSNR values while maintaining a low complexity comparable to that of ML decoding. An additional advantage of the proposed analog JSCC decoding approach is that it can also be used in Multiple Input Multiple Output (MIMO) fading channels.**

***Index Terms*—Analog joint source channel coding, optimum performance theoretically attainable, non-linear mappings, MMSE estimation and decoding.**

I. INTRODUCTION

ANALOG Joint Source Channel Coding (JSCC) has been proposed as an alternative to conventional digital systems based on the separation between source and channel coding [1]. Analog JSCC has been shown [2]–[4] to approach near-optimum performance for high data rates with very low complexity and an almost negligible delay.

Maximum Likelihood (ML) decoding is the most widely used decoding method for analog JSCC [5], [6]. Even though its complexity is very low, it exhibits acceptable performance when the Channel Signal to Noise Ratio (CSNR) is high [3], [5]. However, peformance of ML decoding is rather poor when the CSNR is low. This limitation can be overcome by resorting to Minimum Mean Square Error (MMSE) decoding [7], a method that exhibits an acceptable peformance at all CSNR values. Nevertheless, the complexity of MMSE decoding is significantly larger than that of ML decoding.

In this work, we propose a different decoding approach to analog JSCC that exhibits a performance close to that of MMSE decoding in all CSNR regimes while keeping complexity at the same level as ML decoding. The proposed technique is based on introducing a linear MMSE estimator prior to ML decoding. A similar idea has been discussed in [8] by Forney, who showed that MMSE estimation is instrumental for achieving the capacity of Additive White Gaussian Noise (AWGN) channels but in the context of digital communications using lattice-type coding. In this work, MMSE estimation is also helpful to approach the capacity of fading channels using analog JSCC.

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O. Fresnedo, F. J. Vazquez-Araujo, and L. Castedo are with the Department of Electronics and Systems, University of A Coruña, Spain (e-mail: {ofresnedo, fjvazquez, luis}@udc.es). Their work has been funded by Xunta de Galicia, Ministerio de Economía y Competitividad of Spain, and FEDER funds of the European Union under grants 2012/287, TEC2010-19545-C04-01 and CSD2008-00010.

J. Garcia-Frias is with the Department of ECE, University of Delaware, USA (e-mail: jgarcia@ece.udel.edu). His work has been funded by NSF awards EECS-0725422 and CIF-0915800.

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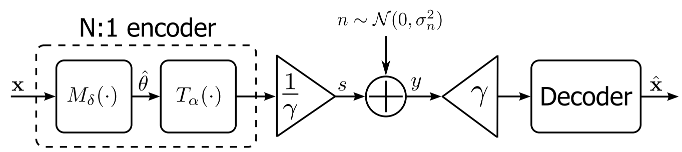

Fig. 1. Block diagram of a bandwidth compression N:1 analog JSCC system over an AWGN channel.

The remainder of this letter is organized as follows. Section II reviews the basics of analog JSCC systems focusing on the limitations of conventional ML and MMSE decoding schemes. Section III describes the proposed low-complexity two-step decoding approach and its application to AWGN, Single Input Single Output (SISO) and Multiple Input Multiple Output (MIMO) fading channels. Section IV presents the results of computer experiments and Section V is devoted to the conclusions.

II. ANALOG JOINT SOURCE-CHANNEL CODING

Figure 1 shows the block diagram of a discrete-time analog JSCC transmission system over an AWGN channel. The system performs an N:1 bandwidth compression, i.e. N analog source symbols are packed into the source vector $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ and compressed into one channel symbol s.

Encoding in analog JSCC typically involves two steps: the compression function $M_\delta(\cdot)$ and the matching function $T_\alpha(\cdot)$. As explained in [5], Shannon-Kotel'nikov mappings can be used to define compression functions $M_\delta(\cdot)$ that map the N source symbols into a single value $\hat{\theta}$. As an example, a particular type of parameterized space-filling continuous curves, called spiral-like curves, can be used to encode the source samples. These curves were proposed for the transmission of Gaussian sources over AWGN channels by Chung and Ramstad [2], [3], [5]. For the case of 2:1 compression (i.e. $N = 2$) the mathematical expression for the two-dimensional spiral is given by

$$\mathbf{z}_\delta(\theta) = \left(\text{sign}(\theta) \frac{\delta}{\pi} \theta \sin\theta, \frac{\delta}{\pi} \theta \cos\theta \right), \quad (1)$$

where δ is the distance between two neighboring spiral arms and θ is the angle from the origin to the point $\mathbf{z} = (z_1, z_2)$ on the curve. For a given spiral-like curve, the compression function $M_\delta(\cdot)$ provides the value $\hat{\theta}$ corresponding to the point on the spiral that minimizes the distance to $\mathbf{x}$, i.e.

$$\hat{\theta} = M_\delta(\mathbf{x}) = \underset{\theta}{\operatorname{argmin}} \|\mathbf{x} - \mathbf{z}_\delta(\theta)\|^2. \tag{2}$$

Next, an invertible function $T_\alpha(\cdot)$ is used to transform the channel symbols. In [2], [3], [5], $T_\alpha(\hat{\theta}) = \hat{\theta}^\alpha$ with $\alpha = 2$ was proposed. However, as shown in [7], system performance can be significantly improved if α is optimized together with δ. We have empirically determined through computer simulations that using $\alpha = 1.3$ provides a good overall performance for 2:1 analog JSCC systems over AWGN channels and a wide range of CSNR and δ values.

Finally, the analog symbol obtained after this transformation is normalized to ensure that the average transmitted power is equal to one. Thus, the channel symbol s is given by

$$s = \frac{T_\alpha(M_\delta(\mathbf{x}))}{\sqrt{\gamma}}, \tag{3}$$

where γ is selected so that $\mathbb{E}[|s|^2] = 1$. When transmitting over an AWGN channel, the received symbol is

$$y = s + n, \tag{4}$$

where $n \sim \mathcal{N}(0, N_0)$ is a real-valued zero-mean Gaussian random variable that represents the channel noise and CSNR $= 1/N_0$.

The aim of decoding is to obtain an estimation of the source symbols, $\mathbf{x}$, from the received symbols, y. The Maximum Likelihood (ML) estimate $\hat{\mathbf{x}}_{\text{ML}}$ is the tuple $(\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N)$ that belongs to the non-linear curve and maximizes the likelihood function $p(y|\mathbf{x})$, i.e.

$$\begin{aligned}\hat{\mathbf{x}}_{\text{ML}} &= \underset{\mathbf{x} \in \text{curve}}{\operatorname{argmax}} \quad p(y|\mathbf{x}) \\ &= \{\mathbf{x} | \mathbf{x} \in \text{curve and } T_\alpha(M_\delta(\mathbf{x}))/\sqrt{\gamma} = y\}.\end{aligned} \tag{5}$$

Particularizing to the case $N = 2$, ML decoding is equivalent to first applying the inverse function T_α^{-1} to the observation y after de-normalization to find an estimate $\tilde{\theta}$ of the transmitted angle $\hat{\theta}$

$$\tilde{\theta} = T_\alpha^{-1}(\sqrt{\gamma} y) = \text{sign}(y) |\sqrt{\gamma} y|^{-\alpha} \tag{6}$$

and then obtaining $\hat{\mathbf{x}}_{\text{ML}} = (\hat{x}_1, \hat{x}_2) = \mathbf{z}_\delta(\tilde{\theta})$. Notice that the overall decoder complexity is extremely low since the two decoding steps previously described only involve simple mathematical operations.

In analog JSCC, system performance is measured in terms of the Signal to Distortion Ratio (SDR) with respect to the CSNR. The distortion is the Mean Square Error (MSE) between decoded and source analog symbols, i.e.

$$\text{MSE} = \frac{1}{N} E\{\|\mathbf{x} - \hat{\mathbf{x}}\|^2\}. \tag{7}$$

The optimal distortion-cost tradeoff is the minimum attainable SDR for a given CSNR. In the literature, this theoretical limit is known as the Optimum Performance Theoretically Attainable (OPTA) and is calculated by equating the rate distortion function to the channel capacity [9].

Analog JSCC over an AWGN channel with ML decoding is analyzed in [2], [3], [5], where it is shown that its performance is close to the OPTA limit for medium and high CSNRs but it significantly degrades if we consider low CSNR values. This unsatisfactory behavior motivates the consideration of Minimum Mean Square Error (MMSE) decoding for analog JSCC, which has been analyzed in [7]. Given an observation y, MMSE decoding calculates the point $\mathbf{x}$ on an N-dimensional space that minimizes the MSE with respect to y, i.e.

$$\begin{aligned}\hat{\mathbf{x}}_{\text{MMSE}} &= \mathbb{E}[\mathbf{x}|y] = \int \mathbf{x}\, p(\mathbf{x}|y) d\mathbf{x} \\ &= \frac{1}{p(y)} \int \mathbf{x}\, p(y|\mathbf{x}) p(\mathbf{x}) d\mathbf{x}.\end{aligned} \tag{8}$$

When MMSE decoding is employed, system performance is close to the theoretical OPTA limit in the whole CSNR region [7]. Nevertheless, calculating $\hat{\mathbf{x}}_{\text{MMSE}}$ is not straightforward. Indeed, since the conditional probability, $p(y|\mathbf{x})$, involves the mapping function $M_\delta(\cdot)$ which is discontinuous and non-linear, the integral in (8) can only be calculated numerically. This implies discretizing the set of all possible source values, $\mathbf{x}$, using a uniform step. If L discrete-points are selected for each source dimension, we have to calculate L^N values for $p(y|\mathbf{x})$ and $p(\mathbf{x})$ and then compute the integral in (8). Although the discretized version of $p(\mathbf{x})$ and the corresponding coded values $T_\alpha(M_\delta(\mathbf{x}))$ can be calculated once off-line and the result stored at the decoder, for MMSE decoding to perform close to the OPTA large values of L have to be chosen, which negatively impacts the decoding complexity.

III. Proposed Decoding Approach

The reason why ML decoding performance degrades at low CSNR is because it produces estimates of the source symbols directly from the received symbols. When the channel noise is high, received symbols are severely distorted and ML estimates are far from the source symbols.

Intuitively, it should be possible to improve the source symbols ML estimates if the received symbols are previously filtered to reduce the distortion that the channel introduces into the transmitted symbols. For this reason, we propose to place an MMSE filter prior to ML decoding with the aim of minimizing the MSE between the transmitted and filtered symbols. In the case of an AWGN channel, the linear MMSE estimate of the transmitted symbols s is given by

$$\hat{s} = \frac{y}{1 + N_0}.$$

Then, ML decoding is applied to the linear filter output $\hat{s}$ and an estimate of the transmitted source symbols is obtained. This two-step decoding strategy resembles the one used in coded-modulated digital systems that makes use of separate detection and decoding stages.

Notice that when transmitting over AWGN channels, the complexity of MMSE filtering is minimum since it simply consists on the multiplication of the channel symbols times a number that depends on N_0. For high CSNRs, this number is close to one and the influence of the MMSE filter is small. However, for low CSNR values the impact of this filtering is significant.

A. SISO fading channels

The proposed two step analog JSCC decoding approach is particularly attractive when considering Single-Input-Single-Output (SISO) fading channels. If a channel symbol s is to be transmitted over a SISO flat-fading channel, the received symbol y is given by

$$y = hs + n \tag{9}$$

where h and n represent the fading channel response and the channel AWGN, respectively. In the case of Rayleigh fading channels, both h and n are modeled as complex-valued zero-mean circularly-symmetric Gaussian independent and identically distributed (i.i.d.) random variables. The fading channel response is normalized (i.e. $\mathbb{E}[|h|^2] = 1$) so that the average CSNR be $1/N_0$.

It is important to note that MMSE analog JSCC decoding is particularly cumbersome when transmitting over fading channels. Although we can pre-calculate the discretized version of $p(\mathbf{x})$ and $T_\alpha(M_\delta(\mathbf{x}))$ off-line, notice that the latter depends on the code parameters α and δ. As previously mentioned, these code parameters have to be optimized for each CSNR value. When transmitting over a fading channel, the CSNR changes at each channel realization. For analog JSCC to perform close to the OPTA, code parameters α and δ have to be continuously adapted to the actual CSNR, which requires many discretized versions of $T_\alpha(M_\delta(\mathbf{x}))$ to be available at the decoder. This causes a significant increase on decoding complexity and storage requirements at the receiver.

In SISO fading channels, the linear MMSE estimate of the transmitted symbol s is given by

$$\hat{s} = \frac{h^* y}{|h|^2 + N_0}, \tag{10}$$

where the super-index * represents complex conjugation. Again, MMSE estimates of the channel symbol, $\hat{s}$, are calculated and subsequently used to obtain estimates of source symbols by ML decoding.

In flat-fading channels, filtering is again a simple scaling and the complexity of this operation is minimum. Notice, however, that the scale factor depends not only on the channel noise variance but also on the channel response. The role of MMSE filtering is particularly important in fading channels where poor channel realizations often cause the CSNR to have small values.

B. MIMO fading channels

The proposed two step analog JSCC decoding approach can also be used when transmitting over Multiple Input Multiple Output (MIMO) fading channels with n_T transmit and $n_R \geq n_T$ receive antennas. In this case, we assume the source symbols are spatially multiplexed over the n_T transmit antennas. At each transmit antenna, i, a set of N analog source symbols is encoded into a channel symbol s_i, $i = 1, \cdots, n_T$ using the encoding procedure described in Section II. It is worth noticing that combining analog JSCC with MIMO transmission provides an overall bandwidth compression ratio Nn_T:1 which can be significantly larger than that obtained with SISO transmissions. At the same time, notice the extremely high complexity of direct MMSE decoding, as defined by (8), which requires the discretization of an Nn_T-dimensional space. If we use L points to discretize each dimension, the total number of points will be L^{Nn_T} which makes MMSE decoding impractical even for moderate values of N and n_T.

When transmitting a channel symbol vector over a frequency flat MIMO fading channel, modeled by an $n_R \times n_T$ channel matrix $\mathbf{H}$ whose entries h_{ij} are random variables, the observed symbol vector at the MIMO channel output can be expressed as

$$\mathbf{y} = \frac{1}{\sqrt{n_T}} \mathbf{H}\mathbf{s} + \mathbf{n} \tag{11}$$

where $\mathbf{s}$, $\mathbf{y}$ and $\mathbf{n}$ are the vectors that represent the channel symbols, the received symbols and the AWGN, respectively. In addition to complex-valued zero-mean circularly-symmetric and Gaussian, the noise is spatially white. Notice that channel symbols are normalized by n_T to ensure unit transmitted power. We also assume a normalized MIMO channel where $\mathbb{E}[|h_{ij}|^2] = 1$. This way the average CSNR is $1/N_0$.

For MIMO channels, the linear MMSE spatial filter that minimizes the MSE between the channel symbol vector $\mathbf{s}$ and the estimated symbol vector $\hat{\mathbf{s}} = \mathbf{W}\mathbf{y}$ is given by

$$\mathbf{W}_{\text{MMSE}} = \left(\mathbf{H}^H \mathbf{H} + n_T N_0 \mathbf{I}_{n_T}\right)^{-1} \mathbf{H}^H \tag{12}$$

The MMSE filter $\mathbf{W}_{\text{MMSE}}$ can be interpreted as an spatial filter that diagonalizes (equalizes) the MIMO channel. Equivalently, the role of $\mathbf{W}_{\text{MMSE}}$ is to transform the MIMO channel into a series of parallel SISO channels where interferences among components in $\hat{\mathbf{s}}$ have been minimized. Thus, components in $\hat{\mathbf{s}}$ are appropriate inputs to ML decoders that produce an estimate of the source symbol vector.

IV. Results

In this section we present the results of several computer experiments carried out to illustrate the performance of the proposed analog JSCC decoding approach. We consider the transmission of Gaussian source samples over three types of channels: AWGN, SISO Rayleigh fading and MIMO spatially white Rayleigh fading. Performance is measured in terms of SDR with respect to CSNR and the obtained results are shown in Figures 2 to 4. The proposed decoding approach is compared to that of ML and MMSE decoding when possible. The OPTA theoretical limit is also plotted as a benchmark. For a generic N:1 MIMO fading $n_T \times n_R$ channel, the OPTA is obtained by equating the SDR to the chanel ergodic capacity, i.e.

$$\frac{1}{N}\log\left(\frac{1}{\text{MSE}}\right) = \frac{1}{n_T}\mathbb{E}_{\mathbf{H}}\left[\log\det\left(\mathbf{I}_{n_R} + \frac{\text{CSNR}}{n_T}\mathbf{H}\mathbf{H}^H\right)\right],$$

where $\mathbb{E}_{\mathbf{H}}[\cdot]$ represents expectation with respect to $\mathbf{H}$. It is straightforward to particularize this equation to obtain the OPTA of a SISO AWGN and a SISO fading channel.

Let us discuss the obtained results in detail. When considering an AWGN channel (see Figure 2) the proposed analog JSCC decoding approach performs similarly to MMSE decoding and both lie within 1.5 dB from the OPTA limit at all CSNR values. Figure 2 also shows how the proposed decoding

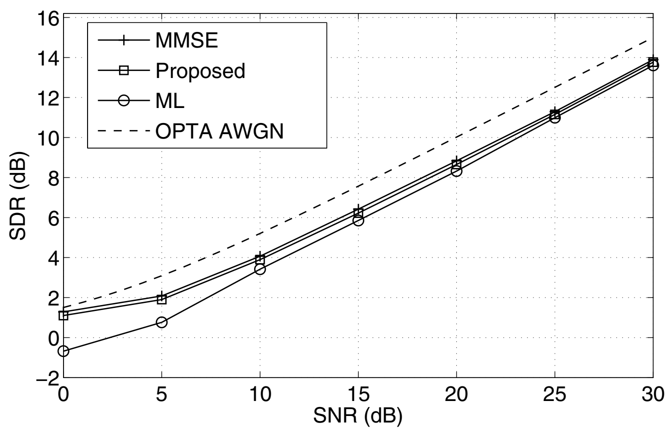

Fig. 2. Performance of the 2:1 analog JSCC system described in the text when an AWGN channel is considered.

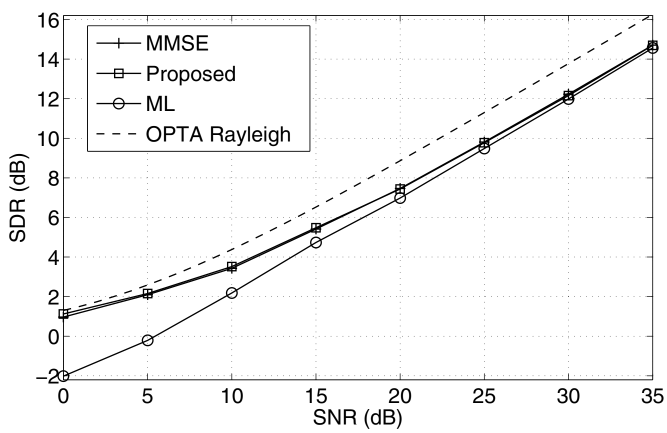

Fig. 3. Performance of the 2:1 analog JSCC system described in the text when a SISO Rayleigh fading channel is considered.

approach outperforms ML decoding at low CSNR values (i.e. below 15 dB).

Similar results were obtained when considering a SISO Rayleigh fading channel as can be seen from Figure 3. The proposed decoding approach and MMSE decoding perform identically and both lie within 2 dB from the OPTA limit at all CSNR values. ML decoding also approaches the OPTA limit for CSNR values above 25 dB, but for lower CSNR values it suffers a significant performance degradation with respect to the proposed scheme and to MMSE decoding.

Finally, Figure 4 shows the ability of the proposed analog JSCC decoding method to approach the OPTA limit when transmitting over 2×2 and 4×4 MIMO Rayleigh fading channels. No curves for ML and MMSE decoding are presented since these decoding methods are unfeasible over MIMO channels. Notice that the asymptotic high CSNR performance of the proposed decoding method is 3 dB below the OPTA limit for the 2×2 MIMO Rayleigh channel. The distance to the OPTA limit in the high CSNR regime increases (up to 4 dB) when considering a MIMO channel with a larger number of antennas (4×4).

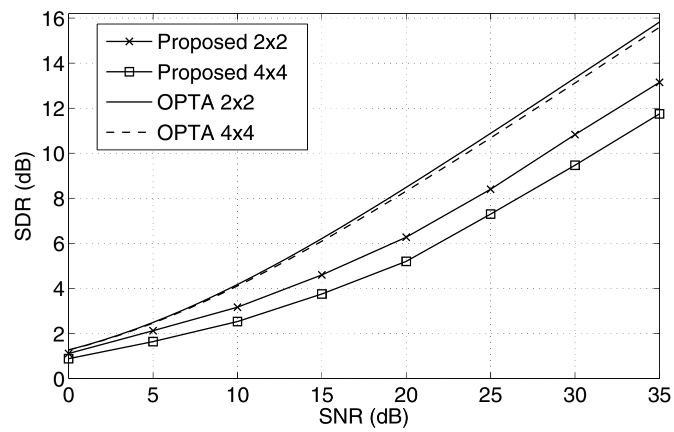

Fig. 4. Performance of the 2:1 analog JSCC system described in the text when 2×2 and 4×4 MIMO Rayleigh fading channels are considered.

V. Conclusions

We have proposed a low-complexity and near-optimal approach to analog JSCC decoding that overcomes the limitations of ML and MMSE decoding methods employed in previous work. The proposed approach is based on the idea of MMSE filtering the received symbols prior to their ML decoding. Computer simulations show that this method performs similarly to MMSE decoding and approaches the OPTA limit at all CSNR values. At the same time, it exhibits very low complexity, similar to that of ML decoding. Another advantage of the proposed method is that it can be applied to MIMO channels where conventional ML and MMSE decoding are unfeasible. Computer simulations show the ability of the proposed approach to perform close to the OPTA limit when considering MIMO spatially white Rayleigh fading channels.

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